Arithmetic Algorithms in Different Bases

Addition, Subtraction, Multiplication and Division

Text Chapter 4 – Section 4

The Addition Algorithm

To add in any base – Step 1

- Add the digits in the "ones "column to find the number of "1s".
  - If the number is less than the base place the number under the right hand column.
  - If the number is greater than or equal to the base, express the number as a base numeral. The first digit indicates the number of the base in the sum so carry that digit to the "base" column and write the second digit under the right hand column.

$$\begin{array}{r} 14 \\ +\ 32_{\,5} \end{array}$$

$2 + 4 = 6$

$6 > 5$

$6 = 11_{\,5}$

Place 1 under $4 + 2$ and carry the 1 five to the "fives" column

No in-class assignment problem

Addition Algorithm - continued

Step 2

- Add the digits in the "base" column.
  - If the sum is less than the base enter under the "base" column.
  - If the sum is greater than or equal to the base express the sum as a base numeral. Carry the first digit to the "base squared" column and place the second digit under the "base" column.

Step 3

- Repeat this process to the end.

$$\begin{array}{r} {\scriptstyle 1} \\ 14 \\ +\ 32_{\,5} \\ \hline 1 \end{array}$$

$1+1+3 = 5$

$5 = 10_{\,5}$

Carry the 1 five to the 5^2 column.

Place 0 under the "five" column

The sum is $101_{\,5}$

In-class Assignment 20 - 1

Example of Addition

Find:

$$\begin{array}{r} {\scriptstyle 1} \\ 123 \\ 342_{\,6} \\ \hline 5 \end{array} \qquad \begin{array}{r} 123 \\ 342_{\,6} \\ \hline 05 \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 123 \\ 342_{\,6} \\ \hline 505_{6} \end{array}$$

- $3 + 2 = 5 < 6 \rightarrow$ no "6"s to carry.
- $2 + 4 = 6 \quad 6 = 10_{\,6}$ One 6^2 to carry. 0 to go in the 6 column.
- $1 + 1 + 3 = 5 < 6 \rightarrow$ no 6^2 to carry.

In-class Assignment 20 - 1

The Subtraction Algorithm

Subtraction in any base, *b*

- Start as "1"s column.
  - If the subtraction is possible enter the number under the "1"s column.
  - If the subtraction is not possible borrow 1 of the groups of *b* in the minuend.
  - The one *b* being borrowed is worth *b* in the "1"s column so add *b* to the digit in the minuend in the "1"s column .
- Continue in this manner through the problem

No in-class assignment problem

A Subtraction Problem

$$\begin{array}{r} 352 \\ -124_{7} \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 4\ 9} \\ 3\not{5}\not{2} \\ -124_{7} \\ \hline 5 \end{array}$$

$$\begin{array}{r} {\scriptstyle 4\ 9} \\ 3\not{5}\not{2} \\ -124_{7} \\ \hline 225_{7} \end{array}$$

- $2-4$ can not do.
- Borrow 1 of the 5 "7"s in the minuend and add 7 to the 2 then subtract.
- $2-4 = 2$
- $3-1 = 2$

In-class Assignment 20 - 2

Another Subtraction Problem

$$\begin{array}{r} 1203 \\ -342_5 \\ \hline \end{array} \qquad \begin{array}{r} 1203 \\ -342_5 \\ \hline 1 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ 5} \\ 12\not{0}3 \\ -342_5 \\ \hline 11 \end{array} \qquad \begin{array}{r} {\scriptstyle 0\ 6\ 5} \\ 1\not{2}\not{0}3 \\ -342_5 \\ \hline 311_5 \end{array}$$

The Multiplication Algorithm

To Multiply in any base

- The multiplication algorithm is the same as in base 10. Things you need to remember.
  - Carry groups of the base
  - Indent for each multiplication row
  - Add in the base

Multiplying in Base 3

$$\begin{array}{r} 2011 \\ 212_3 \\ \hline 11022 \end{array} \qquad \begin{array}{r} 2011 \\ 212_3 \\ \hline 11022 \\ 2011 \end{array} \qquad \begin{array}{r} 2011 \\ 212_3 \\ \hline 11022 \\ 2011 \\ 11022 \\ \hline \end{array} \qquad \begin{array}{r} 2011 \\ 212_3 \\ \hline 11022 \\ 2011 \\ 11022 \\ \hline 1211102_3 \end{array}$$

$2 \times 2 = 4 = 11_3$

Remember to indent

Add in base 3

Carrying in Multiplication

$$\begin{array}{r} 245 \\ 2_6 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 245 \\ 2_6 \\ \hline 4 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ 1} \\ 245 \\ 2_6 \\ \hline 34 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ 1} \\ 245 \\ 2_6 \\ \hline 534_6 \end{array}$$

$2 \times 5 = 10 = 14_6$

$2 \times 4 = 8$, $8 + 1 = 9$, $9 = 1\,3_6$

$2 \times 2 = 4$, $4 + 1 = 5$, $5 = 5_6$

In-class Assignment 20 - 3

The Division Algorithm

- To divide in any base
  - The division algorithm in any base is the same as in base 10. Things to remember –
    - Digits that are entered in the quotient are at first estimates.
    - Do all the multiplications on the side so that the digits used are not estimates.
    - Multiply and subtract in the base used in the problem.

No in-class assignment problem

A Division Problem

$$4_5\overline{)13202_5}\ \begin{array}{r} 2 \\ -\ 13 \\ \hline 0 \end{array} \qquad 4_5\overline{)13202_5}\ \begin{array}{r} 20 \\ -\ 13 \\ \hline 02 \end{array} \qquad 4_5\overline{)13202_5}\ \begin{array}{r} 202 \\ -\ 13 \\ \hline 020 \\ -\ 13 \\ \hline 2 \end{array} \qquad 4_5\overline{)13202_5}\ \begin{array}{r} 2023 \\ -\ 13 \\ \hline 020 \\ -\ 13 \\ \hline 22 \\ -\ 22 \\ \hline 0 \end{array}$$

$1 \times 4_5 = 4_5$
$2 \times 4_5 = 13_5$
$3 \times 4_5 = 22_5$
$4 \times 4_5 = 31_5$

$0 \times 4_5 = 0$

Subtract in base 5

$3 \times 4_5 = 22_5$

In-class Assignment 20 - 4