Numerals in Different Bases

Text Chapter 4 – Section 3

Using Familiar Multipliers

- The multipliers such as ,  , and  , etc. will be replaced by familiar multipliers (digits) 4, 1, and 3, etc.
- Write the numeral below using digits.

Base 9

3 0 2 6_9

No in-class assignment problem

How Many Digits Needed in a Base

- Any natural number greater than 1 may be the base of a numeration system.
- A position numeration system with base, *b*, must have *b* multipliers, digits starting with 0, 1, 2, ..., (*b* – 1).

Example: Digits of base 7: 0, 1, 2, 3, 4, 5, 6.
Digits of base 2: 0, 1.

In-class Assignment 19 - 1

Digits for Bases Greater Than 10

- Each multiplier (digit) must be a single symbol.
- Use A, B, C, D, E, F, etc. as the digits respectively for 10, 11, 12, 13, 14, 15, etc.
- The digits of base 13:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C

- Base 13 has 13 digits.

In-class Assignment 19 - 1

Position Values in Other Bases

- The position (place) values in a numeral are the values of the powers of the base.
- The place value to the extreme right is always 1.
- If the numeral consists of *n* digits then the place value to the extreme left is the base raised to the (*n* –1) power.
- Example: The position values for 52301_8
- are from left to right 8^4, 8^3 , 8^2 , 8^1, 8^0 .

In-class Assignment 19 - 2

Converting a Numeral in a Different Base to Base 10

- In every base the symbol "10" stands for the base. That is 10_{12} = 12, 10_5 = 5, etc.
- Above each digit in the numeral of a given base place the value of the powers of the given base starting at the extreme right and moving to the left.

Place value:	4096	512	64	8	1
Face value:	5	2	3	0	1_8

No in-class assignment problem

Converting a Numeral in a Different Base to Base 10 - continued

Place value: 4096 512 64 8 1

Face value: 5 2 3 0 1_8

- Remember in a positional numeration system to multiply the multiplier (digit) by the place value.
- (5 x 4096) + (2 x 512) + (3 x 64) + (8 x 0) + (1 x 1)
- 20480 + 1024 + 192 + 0 + 1 = 21697
- Therefore, $52301_8 = 21697_{10}$

In-class Assignment 19 - 3

Writing a Base 10 Numeral in Another Base

- Determine the highest power of the new base that is less than or equal to the base 10 numeral.
- Divide the base 10 numeral by the value of that power of the new base.
- Divide the remainder by the next lower power of the new base.
- Continue until only "ones" remain.

No in-class assignment problem

Write 247 as a Base 4 Numeral

- The digits in base 4 are 0, 1, 2, 3.
- Powers of 4: $4^0=1$, $4^1=4$, $4^2=16$, $4^3=64$, $4^4=256 \rightarrow 256 > 247$
- The position values will be from left to right: 64 16 4 1
- How many "64"s are in 247? There are 3.
- How many "16"s in the remainder 55? There are 3. How many "4"s in the remainder 7? There is 1.
- How many "1"s in the remainder 3? There are 3.
- Therefore, $247_{10} = 3313_4$

In-class Assignment 19 - 4

Converting to Another Base

- Things to be aware of.
  - Use only the digits available in the new base.
  - Divide by the powers of the new base.
    - Division is rapid subtraction. So subtraction may be used in place of division.
  - When converting a numeral from a larger base to a smaller base the resulting numeral will look larger even though they are numerals for the same number.
  - When converting a numeral from a smaller base to a larger base the resulting numeral will look smaller.

No in-class assignment problem.

To Convert a Numeral From One Strange Base to Another

- Convert the given numeral in the given base to a base 10 numeral.
  - Need to know the position value of the given base numeral
  - Multiply and add to find the base 10 numeral.
- Convert this base 10 numeral to the other base. Need to know the digits and position values in the new base.
  - Need to know the digits available and position values of this base.
  - Divide to find the final base numeral.

No in-class assignment problem.

Convert 201221_3 to Base 11

Step 1

- Place values: 243 81 27 9 3 1
- Face values: 2 0 1 2 2 1
- (2 x 243)+(0 x 81)+(1 x 27)+(2 x 9)+(2 x 3)+(1 x 1)
- 486 + 0 + 27 + 18 + 6 + 1
- Therefore, $201221_3 = 538_{10}$

No in-class assignment problem

Convert 201221_3 to Base 11
Now 538_{10} to Base 11

Step 2

- Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A
- Powers of 11: 1, 11, 121, next is too large.
- How many "121"s are in 538? 4 remainder 54
- How many "11"s are in 54? 4 remainder A (10)
- Therefore: $201221_3 = 538 = 44A_{11}$

In-class Assignment 19 - 5