

Compound Propositions

Compound Propositions

Conjunction, Disjunction, Conditional and Bi-conditional

Text Chapter 3 – Sections 1, 2

Conjunction

- A conjunction is a compound proposition which consists of two propositions joined by the connective “and” (but, however, also). Denoted $p \wedge q$.
 - 9 is divisible by 3 and 4 is an odd number.
 - $2 + 5 = 10$ but 16 is a multiple of 3.
- A proposition is either true or false. It cannot be both.
- So under what conditions is a conjunction true?

No in-class assignment problem

Truth Values for Conjunctions

You decide!

- Stryket Rich won a TV set and \$1000. He was overheard telling about his winnings to 4 people.
- Decide to whom he told the truth.
 - Jane: I won a TV and \$1000.
 - John: I won a TV and a video camera.
 - Ed: I won a car and \$1000.
 - Mary: I won a car and a video camera.

In-class Assignment 9 - 1

Truth Table for Conjunctions

- Need a heading row and 4 more rows for each possible combination (condition) of true and false.
- Need a column for each proposition and one for the conjunction.
- A conjunction is true only if both propositions are true.

| | p | q | $p \wedge q$ |
|----|---|---|--------------|
| 1. | T | T | T |
| 2. | T | F | F |
| 3. | F | T | F |
| 4. | F | F | F |

In-class Assignment 9 - 1

Disjunction

- A disjunction is a compound proposition which consists of 2 propositions joined by the connective “or.” Denoted $p \vee q$.
- “Or” has two meanings in our language. Use the inclusive or which is the legal and/or type.

No in-class assignment problem

Truth Values of a Disjunction

You Decide.

- John bragged that he could roll a 2 or a 5 when rolling a die twice. He rolled his die twice with the following results for 4 people.
 - Ida: He rolled a 2 and then a 5.
 - Sean: He rolled a 2 and then a 6.
 - Mary: He rolled a 6 and then a 5.
 - Harry: He rolled a 6 and then a 4.
- In which of the cases was John’s brag true?

In-class assignment 9 - 2

Compound Propositions

Truth Table for the Disjunction

- Make the table as for the conjunction.
- A disjunction is false only if both propositions are false.

| p | q | $p \vee q$ |
|------|---|------------|
| 1. T | T | T |
| 2. T | F | T |
| 3. F | T | T |
| 4. F | F | F |

In-class Assignment 9 - 2

Conditional or Implication

- A conditional proposition is a compound proposition which consists of 2 propositions joined by the connective "If ...then ...". Denoted $p \rightarrow q$.
- $p \rightarrow q$ is read "If p then q" or "p implies q."

No in-class assignment problem

Truth Values of a Conditional Proposition – You Decide

- Suppose the first day a class I made you the promise that if you studied hard then I would give you an A.
- Decide when I kept my promise.
 - You studied hard and you got an A.
 - You studied hard but didn't get an A.
 - You didn't study hard and got an A.
 - You didn't study hard and you didn't get an A.

In-class Assignment 9 - 3

Truth Table for Conditionals

- Make the truth table in the same way as for the conjunction and disjunction.
- A conditional is false only when $T \rightarrow F$.

| p | q | $p \rightarrow q$ |
|------|---|-------------------|
| 1. T | T | T |
| 2. T | F | F |
| 3. F | T | T |
| 4. F | F | T |

In-class Assignment 9 - 3

Bi-conditional or Equivalence

- A bi-conditional proposition is a compound proposition which consists of 2 propositions joined by the connective phrase "if and only if."
- Denote: $p \leftrightarrow q$.
Meaning: $(p \rightarrow q) \wedge (q \rightarrow p)$
- It is read as "p if and only if q."
- The word equivalence implies the truth value is true if the propositions have the same truth value.

In-class Assignment 9 - 4

Truth Tables for Bi-conditionals

- Make the table in the same way as for the others.
- A bi-conditional (equivalence) is true only if the truth values are the same.

| p | q | $p \leftrightarrow q$ |
|------|---|-----------------------|
| 1. T | T | T |
| 2. T | F | F |
| 3. F | T | F |
| 4. F | F | T |

In-class Assignment 9 – 4, 5

Compound Propositions

Rules of Logic

- ❏ Negation – change the truth value.
- ❏ Conjunction – True only if both are true.
- ❏ Disjunction – False only if both are false or true if at least one is true.
- ❏ Conditional – False only if true points to false.
- ❏ Bi-conditional – True if the truth values are the same.

No in-class assignment problem

Truth Table for Logic Rules

| | p | q | $p \wedge q$ | $p \vee q$ | $p \rightarrow q$ | $p \leftrightarrow q$ |
|----|---|---|--------------|------------|-------------------|-----------------------|
| 1. | T | T | T | T | T | T |
| 2. | T | F | F | T | F | F |
| 3. | F | T | F | T | T | F |
| 4. | F | F | F | F | T | T |

In-class Assignment 9 - 5