

LOGIC

Propositions and Negations

Text – Chapter 3 – Section 1

PROPOSITIONS - STATEMENTS

- ◉ A proposition is a statement that has the property of being true or false - not both
 - Example 1: 2 is an odd number.
 - Example 2: John went to the store.
- ◉ Questions, commands and exclamations are not considered propositions or statements.
 - Example 1: How are you?
 - Example 2: Close the window.
 - Example 3: Watch out!

In-class Assignment 8 - 1

Recognizing Propositions

- ◉ Which are propositions? - explain
 - $2 + 2 = 4$.
 - Are you ill?
 - Today is Thursday and it is raining.
 - $7 \times 8 = 13$.
 - The sky is blue or John went to school.
 - $8 - 3$
 - Open the window and close the door.
 - What a beautiful day!
 - It is 2 O'clock.

In-class Assignment 8 - 1

SIMPLE PROPOSITIONS

- ◉ A simple proposition is a statement that is singular in nature and conveys one thought.
 - $2 + 5$ is 7.
 - 6 is an odd number.
 - The flower is red.
- ◉ “John and Mary gave a party” is not a simple proposition because it is not singular in nature.
- ◉ “The boy is 10 and his friend is 9” is not a simple proposition because it conveys 2 thoughts.

In-class Assignment 8 - 2

SIMPLE OPEN PROPOSITIONS

- ◉ A simple open proposition is a sentence that contains variables (pronouns) and it will be a proposition when replacements are made for the variables.
 - Example 1: “He went to the store” contains the variable “he”. When “he” is replaced by say “John” then the sentence becomes a proposition.
 - Example 2: $x > 4$, x is a mathematical pronoun and when it is replaced by say 1 then the sentence becomes a proposition.

In-class Assignment 8 - 3

Recognizing Simple and Open Propositions

- ◉ Determine which of the following are simple, open or simple open propositions. Explain.
 - Today is Friday.
 - $x + 7$
 - If it rains then they will go on a picnic.
 - He did it!
 - She went to school on Thursday.
 - Dick and Jane are going to the movie.
 - $2 + 4 < 7$.
 - $X < 0$.

In-class Assignment 8 - 3

NOTATION OF SIMPLE PROPOSITIONS

- Simple propositions are denoted by lower case letters p , q , r , s , t , etc.
- This allows reference to a proposition without saying or writing the entire proposition.
 - Example 1: p : Washington was the first president of the United States.
 - p is true.
 - Example 2: q : $2 + 8 = 11$.
 - q is false.

No in-class assignment problem

Compound propositions

- A compound proposition is a sentence which contains several simple propositions connected by connectives such as “and”, “or”, “although”, “if and only if”, etc.
- The simple components of a compound proposition are denoted by lower case letters p , q , r , s , etc.
 - Today is Friday and it is very hot.
 p and q
- “and” is the connective.

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NEGATION OF A SIMPLE PROPOSITION

- The negation of a proposition is a proposition which changes the truth value of the given proposition.
- The notation for the negation of a given proposition, p , is $\neg p$.
- Simple propositions are usually negated by either the insertion or deletion of the word “not.”
 - Example 1: p : The car is a Ford.
 $\neg p$: The car is not a Ford.
 - Example 2: q : Jane is not a good student.
 $\neg q$: Jane is a good student.

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Negating a Compound Proposition

- A statement that contains more than one thought is called a compound proposition.
- To negate a compound proposition place one of the following in front of the proposition.
 - It is not true that p .
 - It is false that q .
 - It is not the case that r .
 - Example 1: p : 2 is odd and 4 is even.
 $\neg p$: It is not true that 2 is odd and 4 is even.
 - Example 2: q : If I study hard I will succeed.
 $\neg q$: It is false that if I study hard I will succeed.

In-class Assignment 8 - 6

Negating propositions.

- Negate each of the following.
 - The book is heavy.
 - It is not raining.
 - 3 divides 9 and 12 is a multiple of 3.
 - The sky is gray.
 - The division problem is difficult.
 - The rose is red.
 - John knows how to read but he can't do math.
 - If an object has 3 sides then it is a square.
 - Today is Friday or tomorrow is Wednesday.

In-class Assignment 8 - 5, 6

Quantified Propositions and Their Negations

- There are two basic types of quantifiers.
 - The universal quantifier “All”
 - The existential quantifier “Some”
- Negate an “all” statement by changing the quantifier to “some” and negating the rest of the proposition.
 - p : All cats purr. $\neg p$: Some cats do not purr.
- Negate a “some” statement by changing the quantifier to “all” and negating the rest of the proposition or changing “some” to “no”.
 - q : Some cars are blue.
 $\neg q$: All cars are not blue. Or No cars are blue.

In-class Assignment 8 - 7

Negating Quantified Propositions

- ⊙ All and some are not and some and no are negations of each other.
- ⊙ Negate each of the following. Explain.
 - All math is easy.
 - Some dogs have 3 legs.
 - Some classes are not boring.
 - No persons are hungry.
 - All speeders lose their licenses.
 - His mother did not spank him soundly.
 - Some people are lazy.
 - The book fell to the floor.

In-class Assignment 8 - 7

Negations and Truth Tables

- ⊙ A negation, $\neg p$, is true or it is false.
- ⊙ Its truth table will have 2 lines (conditions) beside the heading and 2 columns.
 - A column is needed for p and a column is needed for $\neg p$.
 - p is true or false.
 - The truth value of $\neg p$ must change the truth value of p .

p	$\neg p$
T	F
F	T

No in-class assignment problem