Infinite Sets

Text Chapter 2 – Section 6

Definition of an Infinite Set

- An infinite set is a set whose elements can not be counted.
- An infinite set is one that has no last element.
- An infinite set is a set that can be placed into a one-to-one correspondence with a proper subset of itself.

No in-class assignment problem

One-To-One Correspondence

- A 1-1 correspondence between two sets A and B is a rule that associates each element of set A with one and only one element of set B and vice versa.
- Two sets can be put into a 1-1 correspondence if they have the same cardinal number.

No in-class assignment problem

Example of 1-1 Correspondence

- Let A={x, y, z} and B={5, 6, 7}
- n(A)=3 and n(B)=3
- A 1-1 correspondence is possible.
- There are 6 possible 1-1 correspondences.

A = {x, y, z}
↕ ↕ ↕
B = {5, 6, 7}

In=class Assignment 6 – 1, 2

Contradiction?

- A set is infinite if it can be put into a 1-1 correspondence with a proper subset.
  - 1-1 correspondence says the sets must have the same cardinal number
  - Proper subset says that one set must be smaller in size than the other.

No in-class Assignment problem

An Infinite Set

- E = {2, 4, 6, 8, 10, …, 2n,…}
  - n indicates which number in the sequence is being addressed.
  - If n is 5 then the fifth number in the sequence is 2 times 5 or 10
  - 2n is called the general term.
- F = {4, 8, 12, 16, 20, …,4n,…}
- Is F a proper subset of E?

No in-class assignment problem

The 1-1 Correspondence of Sets E and F

E = {2, 4, 6, 8, 10, …, 2n,…}

↕ ↕ ↕ ↕ ↕ ↕

F = {4, 8, 12, 16, 20, …,4n,…}

- The 16th element of E is 2 times 16 =32
- The 16th element of F is 4 times 16 = 64
- What is the 350th element of each set?
- Since F is a proper subset of E and there is a 1-1 correspondence shown then E is infinite.

In=class Assignment 6 - 3

Countable Sets

- Cantor called the cardinal number of infinite sets "transfinite cardinal numbers."
- A set is countable if it is finite or if it can be placed in a 1-1 correspondence with the set of natural numbers, N = {1, 2, 3, …}.
- A countable set that is infinite has a cardinality of *aleph-null*. The symbol for *aleph-null* is $\aleph_0$.

In-class Assignment 6 – 5, 6.
First statement about cardinal numbers taken from text page 91.