Mathematical Systems

Groups

Chapter 5 – Section 5 and Chapter 10

The Real Numbers

- The real numbers, R, is the set of all unending decimals.
  - If the unending decimal repeats the number belongs to Q and is rational.
    0.5 = 0.5000… = 1/2
    0.33333… = 1/3
  - If the unending decimal does not repeat the number belongs to the set of irrational numbers.
    1.41421…≈ $\sqrt{2}$.
    Symbols are used for irrational numbers.

In-class Assignment 22 -1, 2

Mathematical Systems

- A mathematical system is a set with one or more binary operations defined on it.
  - A binary operation is a rule that assigns to 2 elements of a set a unique third element.
    - If 5 and 7 belong to N and addition is the binary operation then 12 Is the unique "answer."
      5 + 7 = 12
    - If 4 and 4 belong to I and subtraction is the binary operation then 0 is the unique "answer."
      4 – 4 = 0

No in-class assignment problem

Properties of Real Numbers Closure

- If an operation is performed on any two members of the set and the result is a member of the set then the set is closed.
  - If the set is W and the operation is subtraction then 7 − 12 does not belong to W. W is not closed under subtraction.
  - If the set is I and the operation is subtraction then 7 − 12 = −5. This implies I might be closed under subtraction.
- The set R is closed under addition and multiplication. It is not closed under subtraction and division.

In-class Assignment 22 - 3

Properties of Real Numbers Associative Property

- Given 3 numbers of the set in the same order and an operation the result is the same regardless of the grouping.
  (2 + 5) + 4 = 2 + (5 + 4)
  - Notice – order is the same just the grouping is different. 7 + 4 = 2 + 9
    (16 ÷ 8) ÷ 2 ≠ 16 ÷ (8 ÷ 2)

Generally the set R has the associative property under addition and multiplication but not under subtraction and division.

In-class Assignment 22 - 4

Properties of Real Numbers Commutative Property

- Given 2 numbers of a set and an operation the result is the same regardless of the order of the numbers.
  5 + 6 = 6 + 5 but 4 − 2 ≠ 2 − 4
  - Generally the set R has the commutative property under addition and multiplication, but not under subtraction and division.

In-class Assignment 22 - 5

Properties of Real Numbers Identity Property

- A set has the identity property if contains a unique element, *a*, such that the element in operation with any other element in the set in any order results in that number.
- Symbolically: $a + b = b + a = b$
  - $0 + 8 = 8 + 0 = 8$ and $1 \times 15 = 15 \times 1 = 15$
  - Suggests 0 is the identity for addition and 1 is the identity for multiplication.
  - $7 - 0 \neq 0 - 7$ and $9 \div 1 \neq 1 \div 9$
- There is no identity element for subtraction or division.
- The identity property allows the operation to be performed without anything happening.

In-class assignment problem 22 - 6

Properties of Real Numbers Inverse Property

- A set under an operation, $\circ$, has the inverse property if for each element, *a*, of the set there is another element, a^{-1}, (called a inverse) such that $a \circ a^{-1} = a^{-1} \circ a$ = identity.
  $7 + -7 = -7 + 7 = 0$, which is the additive identity
  $\frac{2}{3} \times \frac{3}{2} = 1$, which is the multiplicative identity,
- The inverse property allows for the operation to be undone.

In-class Assignment 22 - 7

Determining Which Properties an Infinite Systems Has - Closure

- E is {2, 4, 6, 8, …} and addition
- Closure –
  - choose 2 numbers
  - Add
  - Does the answer belong to E?
  - Repeat several times.

$6 + 24 = 30$, $30 \in E$

$246 + 12 = 258$, $258 \in E$

E may be closed under addition.

In-class Assignment 22 - 8

Determining Which Properties an Infinite Systems Has - Associative

- E is {2, 4, 6, 8, …} and addition
  - Choose 3 elements of E.
  - Keep same order but different grouping.
  - Are the answers the same?
  - Repeat several times.

$2+(12+8) = 2+20 = 22$
$(2+12)+8 = 14+8 = 22$

$(6+4)+8 = 10+8 = 18$
$6+(4+8) = 6+14 = 18$

E may have the associative property under addition.

In-class Assignment 22 - 9

Determining Which Properties an Infinite Systems Has – Commutative

- E is {2, 4, 6, 8, …} and addition
  - Choose any 2 elements of E.
  - Add
  - Change the order of the elements and add
  - Are the answers the same?
  - Repeat several times.

- $16+24 = 40$
- $24+16 = 40$
- $8+14 = 22$
- $14+8 = 22$
- E may have the commutative property under addition.

In-class Assignment 22 - 10

Determining Which Properties an Infinite Systems Has - Identity

- E is {2, 4, 6, 8, …} and addition
  - Choose any number in E.
  - Try to find another number to add to the chosen number so that the answer is the chosen.
  - An identity must work for all numbers of the set.
  - If no identity then the set can have no inverse property.

$10 + ? = 10$

$344 + ? = 344$

Only 0 would work – so no identity.

E does not have the identity property under addition and no inverse property.

In-class Assignment 22 - 11