Section M
Discrete Probability Distribution

Now that we know something about probability, we can extend the concepts that were discussed earlier in this course, specifically, relative-frequency distribution, mean and standard deviation, which we applied to variables of a finite size to other types of variables. This leads to a discussion on the basics of random variables and their probability distribution.

A **random variable** is a numerical measure of the outcome of a probability experiment, so its value is determined by chance. Random variables are typically denoted using capital letters such as X.

A **discrete random variable** has either a finite or countable number of value; possible values can be listed.

A **continuous random variable** has infinitely many values; possible values cannot be listed.

Example:
1) Determine whether the random variable is discrete or continuous.

a) The number of cars in a parking lot. **Discrete**

b) The time you wait in line at a check out. **Continuous**

c) The height of a building. **Continuous**

d) The number of students in a classroom. **Discrete**

e) The number of times you flip a coin. **Discrete**

f) The weight of a passenger's suitcase. **Continuous**

The **probability distribution** of a discrete random variable X provides the possible values of the random variable and their corresponding probabilities. A probability distribution can be in the form of a table, graph, or mathematical formula.

Rules for a Discrete Probability Distribution

Let P(x) denote the probability that the random variable has the value x. Then

1) $\sum P(x) = 1$ and 2) $0 \le P(x) \le 1$

This means the sum of all the probabilities in a discrete probability distribution must add up to 1 and each individual probability can never be negative or greater than 1.

Example:

2) Determine whether the table represents a discrete probability distribution. If not, explain why.

a)

x	P(x)
10	0.3
20	0.2
30	0.2
40	0.3

Yes

b)

x	P(x)
-2	0.15
-1	0.35
1	0.25
2	0.25

Yes

c)

x	P(x)
6.2	-0.2
6.3	0.4
6.4	0.6
6.5	0.2

No, has a negative probability

d)

x	P(x)
15	0.3
20	0.4
25	0.5
30	0.6

No, probability adds to more than 1

3) Fill in the missing value so that the following table represents a probability distribution.

x	P(x)
5	0.15
10	
15	0.25
20	0.45
25	0.05

0.15 + 0.25 + 0.45 + 0.05 = 0.9 so 1 – 0.9 = 0.1

A probability histogram is a histogram in which the horizontal axis corresponds to the value of the random variable and the vertical axis represents the probability of each value of the random variable. It's the same as a relative frequency histogram.

The **mean** or **expected value** of a discrete random variable

mean of the random variable X = $\boldsymbol{\mu_X = \sum(x \cdot P(x))}$, where *x* is the value of the random variable and P(*x*) is the probability of observing the value *x*.

Note: the mean of a discrete random variable is thought of as the average outcome if the experiment is repeated many, many times. In other words, if a probability experiment that produces a value of a random variable is repeated over and over again, the average of the values produced will approach the mean of the random variable.

Law of Large Numbers If we sample from a population, then as the sample grows larger, the sample mean will approach the population mean.

Variance and **standard deviation** of a discrete random variable

variance of the random variable X = $\boldsymbol{\sigma_X^2 = \sum(x^2 \cdot P(x)) - \mu_X^2}$

standard deviation of the variable X = $\boldsymbol{\sigma_X = \sqrt{\sigma_X^2} = \sqrt{\sum(x^2 \cdot P(x)) - \mu_X^2}}$

Example:

4) Compute the mean and standard deviation of the random variable with the given discrete probability distribution.

a)

x	P(x)
10	0.25
15	0.15
20	0.35
25	0.2
30	0.05

$$\mu_X = \mathbf{10(0.25) + 15(0.15) + 20(0.35) + 25(0.2) + 30(0.05) = 18.25}$$

$$\sigma_X = \sqrt{\mathbf{10^2(0.25) + 15^2(0.15) + 20^2(0.35) + 25^2(0.2) + 30^2(0.05) - 18.25^2}} = \mathbf{5.97}$$

b)

x	P(x)
-2	0.27
-1	0.22
0	0.18
2	0.29
3	0.04

$$\mu_X = \mathbf{-2(0.27) - 1(0.22) + 0(0.18) + 2(0.29) + 3(0.04) = -0.06}$$

$$\sigma_X = \sqrt{\mathbf{(-2)^2(0.27) + (-1)^2(0.22) + 0^2(0.18) + 2^2(0.29) + 3^2(0.04) - (-0.06)^2}} = \mathbf{1.68}$$

5) The number of points scored in a domino tournament on a typical scoring play has the following probability distribution.

x	5	10	15	20	25
P(x)	0.09	0.11	0.30	0.29	0.21

a) What is the probability of scoring 10 or less? **0.09 + 0.11 = 0.20**

b) What is the probability of scoring 20 or more? **0.29 + 0.21 = 0.5**

c) What is the probability of scoring 15? **0.30**

d) What is the mean? $\mu_X = \mathbf{5(0.09) + 10(0.11) + 15(0.3) + 20(0.29) + 25(0.21) = 17.1}$

e) What is the standard deviation?

$$\sigma_X = \sqrt{\mathbf{5^2(0.09) + 10^2(0.11) + 15^2(0.30) + 20^2(0.29) + 25^2(0.21) - 17.1^2}} = \mathbf{5.97}$$

6) The following table defines the discrete distribution for the number of cars per household in California.

Number of Cars	0	1	2	3	4 or more
P(x)	0.03	0.13	0.70	0.10	0.04

a) What is the probability a California household owns 1 car? **0.13**

b) What is the probability a California household owns more than 2 cars? **0.10 + 0.04 = 0.14**

c) What is the probability a California household owns less than 3 cars? **0.03 + 0.13 + 0.70 = 0.86**

d) What is the mean? $\boldsymbol{\mu_X = 0(0.03) + 1(0.13) + 2(0.7) + 3(0.1) + 4(0.04) = 1.99}$

e) What is the standard deviation?

$$\boldsymbol{\sigma_X = \sqrt{0^2(0.03) + 1^2(0.13) + 2^2(0.70) + 3^2(0.1) + 4^2(0.04) - 1.99^2} = 0.71}$$

7) An insurance company sells a one-year term life insurance policy to a 80-year-old woman. The woman pays a premium of $5000. If she dies within one year, the company will pay $50,000 to her beneficiary. According to U.S. Centers for Disease Control and Prevention, the probability that a 80-year-old woman will be alive one year later is 0.9516. Let X be the profit made by the insurance company.

a) Construct a probability distribution.

x	**P(x)**
5000	**0.9516**
-50000	**0.0484**

1 – 0.9516 = 0.0484
Since the sum must add to one.

b) Find the expected value of the profit.

$\boldsymbol{\mu_X = 5000(0.9516) + (-50000)(0.0484) =}$ **$2338**

8) An investor is considering a $20,000 investment in a start-up company. She estimates that she has a probability of 0.20 of a $15,000 loss, probability of 0.35 of a $25,000 profit, probability of 0.15 of a $50,000 profit, and probability 0.30 of breaking even (a profit of $0). What is the expected value of the profit? Would you advise the investor to make the investment? Explain.

x	P(x)
0	0.30
−15000	0.20
25000	0.35
50000	0.15

$$\mu_X = \mathbf{0(0.3) + (-15000)(0.2) + 25000(0.35) + 50000(0.15)} = \$\mathbf{13250}$$

Yes, I would advisor the investor to make the investment since the expected profit is positive.

9) You play a game with an ordinary deck of 52 cards where one card is drawn at random. If the card drawn is the ace of diamonds you win $55. If the card is any diamond other than the ace you win $10. If the card is black, you win $5. However, if you pick a heart, you lose $30.

a) Construct a probability distribution and find the expected value of this game for you.

x	P(x)
55	$\frac{1}{52}$
10	$\frac{12}{52}$
5	$\frac{26}{52}$
-30	$\frac{13}{52}$

$$\mu_X = \mathbf{55}\left(\frac{\mathbf{1}}{\mathbf{52}}\right) + \mathbf{10}\left(\frac{\mathbf{26}}{\mathbf{52}}\right) + \mathbf{5}\left(\frac{\mathbf{26}}{\mathbf{52}}\right) + (\mathbf{-30})\left(\frac{\mathbf{13}}{\mathbf{52}}\right) = \mathbf{-1.63}$$

b) Is it to your advantage to play? Explain.

No, since the expected value for the game is negative.