$$y' = Ky$$

pop growth y' is proportional to it population y

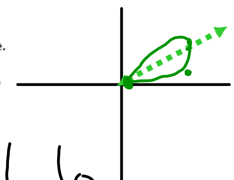
$$P = Qe^{RT}$$

$$P' = Qe^{RT} \cdot R$$

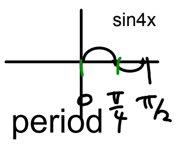
$$Qe^{RT} \cdot R = \underset{\underset{R}{\shortparallel}}{K} \cdot Qe^{RT}$$

Find the area of the region bounded by the given curve.
one leaf of $r = 12 \sin 4\theta$

$A =$ ☐



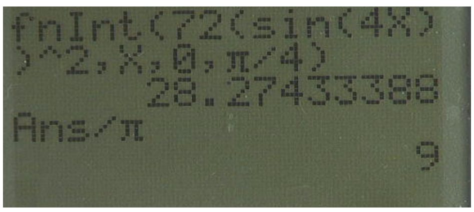
$$A = \int_{\theta=0}^{\pi/4} \int_{r=0}^{12\sin 4\theta} 1\, r\, dr\, d\theta$$

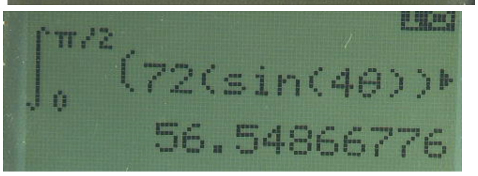


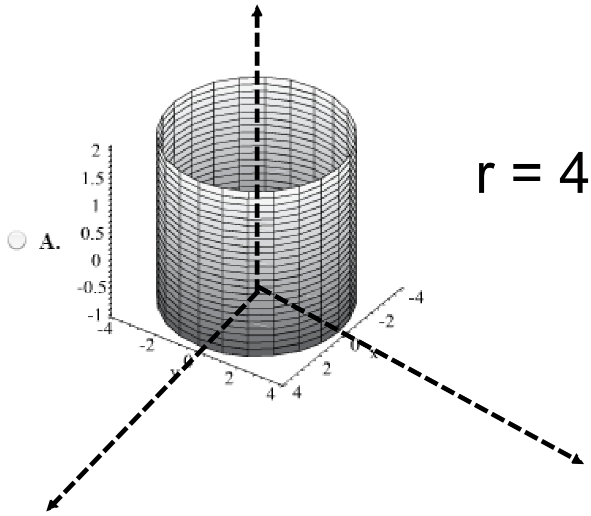
period
=2pi/4=pi/2

$$\left.\frac{r^2}{2}\right|_0^{12\sin 4\theta}$$

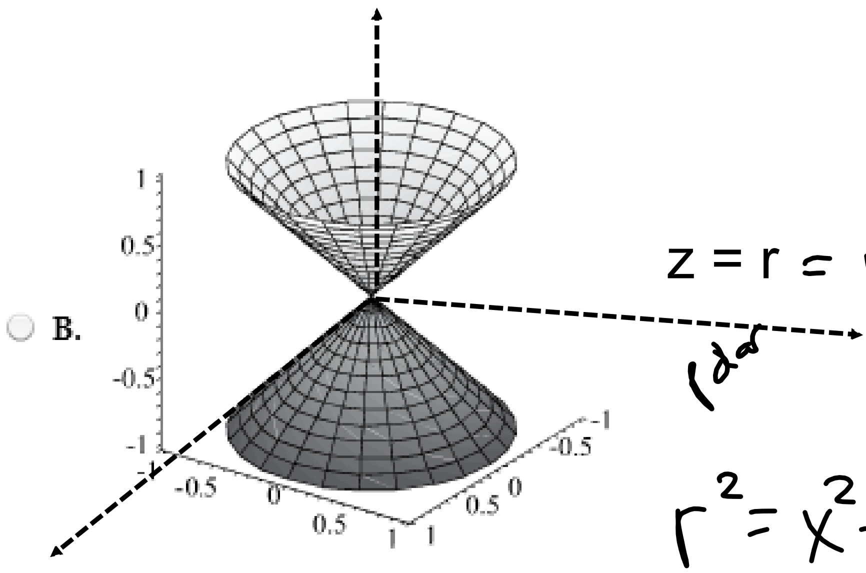
$$\int_0^{\pi/4} 72 \sin^2(4\theta)\, d\theta = 9\pi$$







r = 4

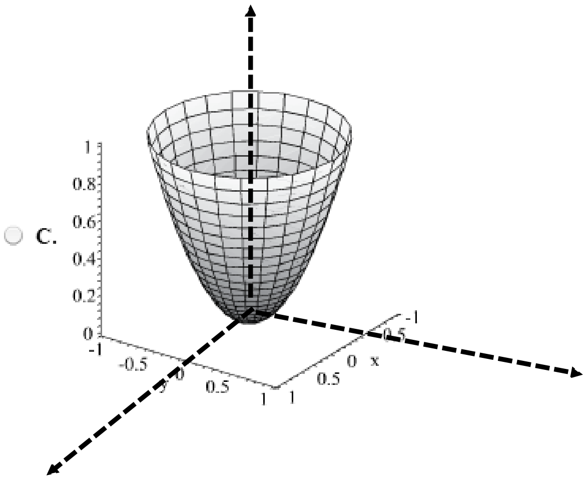


$z = r = \sqrt{x^2 + y^2}$

polar

Rectangular

$r^2 = x^2 + y^2$



$z = r^2$

$z = x^2 + y^2$

Parabaloid

Sperical Coordinats

Rectangle (x, y, z)

cylindrical (r, θ, z)

spherical (ρ, θ, ϕ) phi

$(x, y, z) \longrightarrow \left(\sqrt{x^2+y^2+z^2},\ \tan^{-1}\left(\frac{y}{x}\right),\ \tan^{-1}\left(\frac{\sqrt{x^2+y^2}}{z}\right)\right)$

$\tan\theta = \frac{y}{x}$

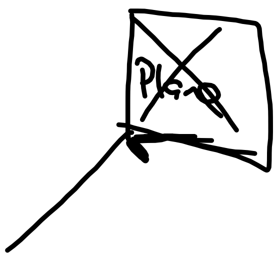
cart $\leftrightarrow$ cyl
$$\begin{cases} x = r\cos\theta, \\ y = r\sin\theta, \end{cases} \qquad \begin{cases} r = \sqrt{x^2+y^2}, \\ \theta = \arctan\dfrac{y}{x}, \end{cases} \qquad \begin{cases} \sin\theta = \dfrac{y}{\sqrt{x^2+y^2}}, \\ \cos\theta = \dfrac{x}{\sqrt{x^2+y^2}}. \end{cases}$$

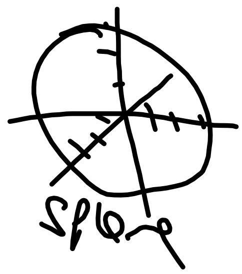
cyl $\leftrightarrow$ sph
$$\begin{cases} r = \rho\sin\phi, \\ z = \rho\cos\phi, \end{cases} \qquad \begin{cases} \rho = \sqrt{r^2+z^2}, \\ \phi = \arctan\dfrac{r}{z}, \end{cases} \qquad \begin{cases} \sin\phi = \dfrac{r}{\sqrt{r^2+z^2}}, \\ \cos\phi = \dfrac{z}{\sqrt{r^2+z^2}}. \end{cases}$$

cart $\leftrightarrow$ sph
$$\begin{cases} x = \rho\cos\theta\sin\phi, \\ y = \rho\sin\theta\sin\phi, \\ z = \rho\cos\phi, \end{cases} \qquad \begin{cases} \rho = \sqrt{x^2+y^2+z^2}, \\ \theta = \arctan\dfrac{y}{x}, \\ \phi = \arctan\dfrac{\sqrt{x^2+y^2}}{z} \\ \quad = \arccos\dfrac{z}{\sqrt{x^2+y^2+z^2}}. \end{cases}$$

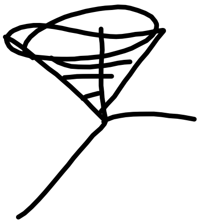
$$\rho = 3 = \sqrt{x^2+y^2+z^2}$$
$$x^2+y^2+z^2 = 9$$

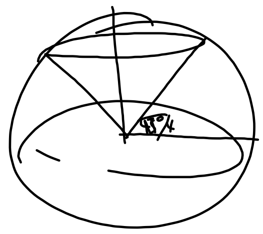
$$\theta = \frac{\pi}{2}$$





$$\phi = \frac{\pi}{4}$$ Cone





$\rho = 3$

$V = \iiint_Q 1\, dV$

$\int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi/4} \int_{\rho=0}^{3} 1\, \underbrace{\rho^2 \sin\phi}\, d\rho\, d\phi\, d\theta$

$\theta = 0 \to 2\pi$ $\quad \int_0^{\pi/4} \sin\phi\, d\phi \quad \int_0^3 \rho^2\, d\rho$

$2\pi \cdot -\cos\phi \Big|_0^{\pi/4} \cdot \frac{\rho^3}{3}\Big|_0^3$

$2\pi \left(-\frac{\sqrt{2}}{2} + 1\right) 9$

$18\pi(1 - \sqrt{2}/2)$

In exercises 1–6, convert the spherical point (ρ, ϕ, θ) into rectangular coordinates.

$\rho\ \phi\ \theta$

1. $(4, 0, \pi)$ 2. $(4, \frac{\pi}{2}, \pi)$ 3. $(2, \frac{\pi}{4}, 0)$

$\downarrow (x, y, z)$

$x = \rho\cos\theta\sin\phi \quad y = \rho\sin\theta\sin\phi \quad z = \rho\cos\phi$

$(4\cos\pi\sin 0,$ $\quad x^2 + y^2 = \rho^2\sin^2\phi + \rho^2\cos^2\phi$

ρ^2

$(0, 0, 4)$

In exercises 7–14, convert the equation into spherical coordinates.

7. $x^2 + y^2 + z^2 = 9$ $\rho^2 = 9$ $\rho = 3$
8. $x^2 + y^2 + z^2 = 6$
9. $y = x$ $\theta = \pi/4$
10. $z = 0$
11. $z = 2$
12. $x^2 + y^2 + (z-1)^2 = 1$
13. $z = \sqrt{3(x^2 + y^2)}$
14. $z = -\sqrt{x^2 + y^2}$

$z = \rho\cos\varphi = 2$

$\rho = 2\sec\varphi$

$x = \rho\cos\theta\sin\varphi$

$y = \rho\sin\theta\sin\varphi$

$\sin\theta = \cos\theta$

$\tan\theta = 1$

$\theta = \tan^{-1}(1) = \frac{\pi}{4}$

$x^2 + y^2 + z^2 - 2z + 1 = 1$

$\rho^2 - 2\rho\cos\varphi = 0$

$\rho - 2\cos\varphi = 0$

$\rho = 2\cos\varphi$

30. $\iiint_Q e^{\sqrt{x^2+y^2+z^2}}\, dV$, where Q is bounded by $y = \sqrt{4 - x^2 - z^2}$ and $y = 0$.

$y^2 + x^2 + z^2 = 4$

$\rho^2 = 4$

$\rho = 2$

$$\int_{\theta=0}^{\pi}\int_{\varphi=0}^{\pi}\int_{\rho=0}^{2} e^{\rho}\,\underbrace{\rho^2\sin\varphi}_{\text{Jacobian}}\, d\rho\, d\varphi\, d\theta$$

$\pi \int_0^{\pi}\sin\varphi\, d\varphi \int_0^2 \rho^2 e^{\rho}\, d\rho$

$(-\cos\pi + \cos 0)$

2π

$+\ \rho^2 \quad e^{\rho}$

$-\ 2\rho \quad e^{\rho}$

$+\ 2 \quad e^{\rho}$

$0 \quad e^{\rho}$

$\left(\rho^2 e^{\rho} - 2\rho e^{\rho} + 2e^{\rho}\right)\Big|_0^2$

$4e^2 - 4e^2 + 2e^2 - 0 - 0 - 2e^0$

$2\pi\left(2e^2 - 2\right)$

ρ^2

36. $\iiint_Q (x^2+y^2+z^2)^{3/2}\,dV$, where Q is the solid below $z=-\sqrt{x^2+y^2}$ and inside $z=-\sqrt{4-x^2-y^2}$.

$z^2=x^2+y^2$ cone

Sphere. rad 2

$$\int_{\phi=\frac{3\pi}{4}}^{\pi}\int_{\theta=0}^{2\pi}\int_{\rho=0}^{2} \rho^3\cdot\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$$

$$\phi=\arctan\frac{\sqrt{x^2+y^2}}{z=-\sqrt{x^2+y^2}}$$

$\tan^{-1}(-1)$

rad $\sqrt{8}$

51. $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}}(x^2+y^2+z^2)^{3/2}\,dz\,dy\,dx$

$y=$ $z=$ cone.

$y=\sqrt{4-x^2}$

$x^2+y^2=4$

$$\int_0^{\pi/4}\int_0^{\pi}\int_0^{2\sqrt2}\rho^3(\rho^2\sin\phi)\,d\rho\,d\phi\,d\theta$$

$0<\theta<\pi$

$\sqrt{x^2+y^2}=\sqrt{8-x^2-y^2}$

$2(x^2+y^2)=8$

$x^2+y^2=4$

$\tan\phi=\frac{\sqrt{x^2+y^2}}{z}$

$\tan\phi=1$

$\phi=\pi/4$