Construct the function $d(x, y)$ giving the distance from a point (x, y, z) on the paraboloid $z = 4 - x^2 - y^2$ to the point $(2, -5, 1)$. Then determine the point that minimizes $d(x, y)$. Round all your intermediate calculations to four decimal places and round your final answer to one decimal place.

The closest point on the paraboloid to the point $(2, -5, 1)$ is approximately

(______, ______, ______).

f = Distance^2 = $(x-2)^2 + (y+5)^2 + (z-1)^2$

g = Constraint: $z = 4 - x^2 - y^2$

$0 = 4 - x^2 - y^2 - z = g$

$\nabla f = \langle 2(x-2), 2(y+5), 2(z-1) \rangle$

$\nabla g = \langle -2x, -2y, -1 \rangle$

$\nabla f = \lambda \nabla g$

$\cdot y \quad 2(x-2) = -2\lambda x$

$2 \qquad x \cdot \quad 2(y+5) = -2\lambda y$

$2(z-1) = -\lambda$

$z = \frac{-\lambda}{2} + 1$

$z = 4 - x^2 - y^2$

$z = 4 - x^2 - \left(\frac{5}{2}x\right)^2$

$z = 4 - x^2 - \frac{25}{4}x^2$

$z = 4 - \frac{29}{4}x^2$

$2y(x-2) = 2x(y+5)$

$\frac{x-2}{x} = \frac{y+5}{y}$

$1 - \frac{2}{x} = 1 + \frac{5}{y}$

$-\frac{2}{x} = \frac{5}{y}$

$y = -\frac{5}{2}x$

$\frac{-\lambda+2}{2} = \frac{16 - 29x^2}{4}$

$-2\lambda + 4 = 16 - 29x^2$

$\lambda = \frac{12 - 29x^2}{-2}$

$\nabla f = \lambda \nabla g$

$\cdot y \quad 2(x-2) = -2\lambda x$

$2 \qquad x \cdot \quad 2(y+5) = -2\lambda y$

$2(z-1) = -\lambda$

$z = \frac{-\lambda}{2} + 1$

$z = 4 - x^2 - y^2$

$z = 4 - x^2 - \left(\frac{5}{2}x\right)^2$

$z = 4 - x^2 - \frac{25}{4}x^2$

$\lambda = \underbrace{\frac{2-x}{x} = \frac{-y-5}{y}}_{y = \frac{5}{2}x} = 2 - 2z$

$\frac{2}{x} - 1 = 2 - 2z$

$\frac{2}{x} - 1 = 2\left(4 - x^2 - \frac{25}{4}x^2\right)$

$2 - x = 8x \underbrace{- 2x^3 - \frac{25}{2}x^3}$

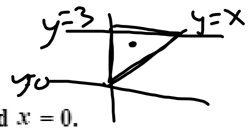
$0 = -\frac{29}{2}x^3 + 9x - 2$

Round your answers to three decimal places, if needed.

Find the absolute extrema of the function

$f(x, y) = 4.2x^2 + 2.4y^2 - 4.5x - 3.1y$

on the region bounded by $y = x$, $y = 3$, and $x = 0$.



The absolute maximum is $\boxed{36.6}$.

The absolute minimum is $\boxed{-2.20}$.

$f_x = 8.2x - 4.5 = 0 \quad x = 4.5/8.2 \approx 1/2 = .549$

$f_y = 4.8y - 3.1 = 0 \quad y = 3.1/4.8 \approx .65$

$f(.549, .65) = 4.2(.549)^2 + 2.4(.65)^2 - 4.5(.549) - 3.1(.65)$

$= -2.20$

$f(0,0) = 0 \quad f(3,3) = 36.6$

$f(0,3) = 12.3$

$x=0 \quad z = 2.4y^2 - 3.1y \quad z' = 4.8y - 3.1 = 0 \quad y = 3.1/4.8 \quad (0, .65) \quad f(0,.65) = -1$

$y=x \quad z = 6.6x^2 - 7.6x \quad z' = 13.2x - 7.6 = 0 \quad x = \frac{7.6}{13.2} \quad f(.57, .57) = -2.18$

$y=3 \quad z = 4.2x^2 - 4.5x + 11 \quad z' = 0 \quad 8.4x = 4.5 \quad x = 4.5/8.4 \quad y = 3$

$f(\ , \ ) = 11.$

Minimize $f(x, y, z) = x^2 + y^2 + z^2$, subject to the constraints $x + 2y + 3z = 6$ and $y + z = 0$.

Your Answer: $\boxed{(4, -2, 2)}$

$f(4,-2,2) = 16 + 4 + 4 = 24$

$\nabla f = \langle 2x, 2y, 2z \rangle$

$\nabla g = \langle 1, 2, 3 \rangle$

$\nabla h = \langle 0, 1, 1 \rangle$

$2x = \lambda$

$2y = 2\lambda + \mu$

$2z = 3\lambda + \mu$

$2z - 2y = \lambda$

$2z + 2z = \lambda$

$4z = \lambda$

$x = 2z$

$y = -z$

$x + 2y + 3z = 6$

$2z - 2z + 3z = 6$

$z = 2$

$x = 4$

$y = -2$

Find the distance between the parallel planes:

$P_1: 2x - 5y + z = 7$

and $P_2: 6x - 15y + 3z = 9.$

point (0,0,3)

$d = \square$

distance from point (0,0,3) to

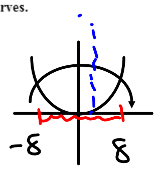
2x - 5y + z = 7

$$\frac{|2(0) - 5(0) + 3 - 7|}{\sqrt{2^2 + 5^2 + 1^2}}$$

$$\frac{4}{\sqrt{30}}$$

Use a double integral to compute the area of the region bounded by the curves.

$y = x^2,\ y = 128 - x^2$



$$\boxed{A = \iint_R 1\, dA}$$

$$\int_{-8}^{8} \int_{x^2}^{128-x^2} dy\, dx$$

$$\int_{-8}^{8} y \Big|_{x^2}^{128-x^2} dx$$

$$\int_{-8}^{8} (128 - x^2) - x^2\, dx$$

$$\int_{-8}^{8} 128 - 2x^2\, dx$$

$$128x - \frac{2x^3}{3} \Big|_{-8}^{8}$$

$$2\left(128(8) - \frac{2(8)^3}{3}\right)$$

$$=(682.6)^2$$

$$= 1365$$

$128 - x^2 = x^2$

$128 = 2x^2$

$64 = x^2$

$\pm 8 = x$

Compute the directional derivative of f at the given point in the direction of the indicated vector.

$f(x, y) = e^{2x^2 - y}$, (1, 2), u in the direction of $-3\mathbf{i} - 2\mathbf{j}$

$u = \frac{\langle -3, -2 \rangle}{\sqrt{13}}$

$D_u f(1, 2) =$ ☐

$\nabla f \cdot u$

$\langle e^{2x^2-y} \cdot 4x, e^{2x^2-y}(-1) \rangle$

$\nabla f(1,2) = \langle 4, -1 \rangle$

$\langle 4, -1 \rangle \frac{\langle -3, -2 \rangle}{\sqrt{13}} = \frac{-10}{\sqrt{13}}$

Minimize $f(x, y, z) = x^2 + y^2 + z^2$, subject to the constraints $x + 2y + 4z = -18$ and $y + z = 0$.

Your Answer: $3 \cdot 36 = 108$

$(-6, 6, -6)$

$\nabla f = \langle 2x, 2y, 2z \rangle$

$\nabla g = \langle 1, 2, 4 \rangle$

$\nabla h = \langle 0, 1, 1 \rangle$

$2x = \lambda$

$2y = 2\lambda + \mu$

$2z = 4\lambda + \mu$

$x + 2y + 4z = -18$

$z + -2z + 4z = -18$

$3z = -18$

$z = -6$

$2\lambda = 2z - 2y$

$2\lambda = 2z + 2z$

$2\lambda = 4z$

$\lambda = 2z$

$2x = 2z$

$x = z$

$y = -z$