EXERCISES 13.7

WRITING EXERCISES

1. Discuss the relationship between the spherical coordinates angles ϕ and θ and the longitude and latitude angles on a map of the earth. Satellites in geosynchronous orbit remain at a constant distance above a fixed point on the earth. Discuss how spherical coordinates could be used to represent the position of the satellite.

2. Explain why any point in $\mathbb{R}^3$ can be represented in spherical coordinates with $\rho \geq 0, 0 \leq \theta \leq 2\pi$ and $0 \leq \phi \leq \pi$. In particular, explain why it is not necessary to allow $\rho < 0$ or $\pi < \phi \leq 2\pi$. Discuss whether the ranges $\rho \geq 0, 0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$ would suffice to describe all points.

3. For simplicity, we restricted ρ to be nonnegative. Discuss what might be meant by spherical coordinates $\rho = -1, \phi = \frac{\pi}{4}$ and $\theta = \frac{\pi}{2}$. Discuss possible advantages of such a definition for graphing functions $\rho = f(\phi, \theta)$.

4. Using the examples in this section as a guide, make a short list of surfaces that are simple to describe in spherical coordinates.

In exercises 1–6, convert the spherical point (ρ, ϕ, θ) into rectangular coordinates.

1. $(4, 0, \pi)$
2. $(4, \frac{\pi}{2}, \pi)$
3. $(2, \frac{\pi}{4}, 0)$
4. $(2, \frac{\pi}{4}, \frac{2\pi}{3})$
5. $(\sqrt{2}, \frac{\pi}{6}, \frac{\pi}{3})$
6. $(\sqrt{2}, \frac{\pi}{6}, \frac{2\pi}{3})$

In exercises 7–14, convert the equation into spherical coordinates.

7. $x^2 + y^2 + z^2 = 9$
8. $x^2 + y^2 + z^2 = 6$
9. $y = x$
10. $z = 0$
11. $z = 2$
12. $x^2 + y^2 + (z - 1)^2 = 1$
13. $z = \sqrt{3(x^2 + y^2)}$
14. $z = -\sqrt{x^2 + y^2}$

In exercises 15–20, sketch the graph of the spherical equation and give a corresponding *xy*-equation.

15. $\rho = 2$
16. $\rho = 4$
17. $\phi = \frac{\pi}{4}$
18. $\phi = \frac{\pi}{2}$
19. $\theta = 0$
20. $\theta = \frac{\pi}{4}$

In exercises 21–26, sketch the region defined by the given ranges.

21. $0 \leq \rho \leq 4, 0 \leq \phi \leq \frac{\pi}{4}, 0 \leq \theta \leq \pi$
22. $0 \leq \rho \leq 4, 0 \leq \phi \leq \frac{\pi}{2}, 0 \leq \theta \leq 2\pi$
23. $0 \leq \rho \leq 3, \frac{\pi}{2} \leq \phi \leq \pi, 0 \leq \theta \leq \pi$
24. $0 \leq \rho \leq 3, 0 \leq \phi \leq \frac{3\pi}{4}, \frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}$
25. $2 \leq \rho \leq 3, \frac{\pi}{4} \leq \phi \leq \frac{\pi}{2}, \pi \leq \theta \leq 2\pi$
26. $2 \leq \rho \leq 3, \frac{\pi}{2} \leq \phi \leq \frac{3\pi}{4}, 0 \leq \theta \leq \frac{3\pi}{2}$

In exercises 27–36, set up and evaluate the indicated triple integral in an appropriate coordinate system.

27. $\iiint_Q e^{(x^2+y^2+z^2)^{3/2}} dV$, where Q is bounded by the hemisphere $z = \sqrt{4 - x^2 - y^2}$ and the xy-plane.
28. $\iiint_Q \sqrt{x^2 + y^2 + z^2}\, dV$, where Q is bounded by the hemisphere $z = -\sqrt{9 - x^2 - y^2}$ and the xy-plane.
29. $\iiint_Q z^2 dV$, where Q is inside $x^2 + y^2 + z^2 = 2$ and outside $x^2 + y^2 = 1$.
30. $\iiint_Q e^{\sqrt{x^2+y^2+z^2}} dV$, where Q is bounded by $y = \sqrt{4 - x^2 - z^2}$ and $y = 0$.
31. $\iiint_Q (x^2 + y^2 + z^2)\, dV$, where Q is the cube with $0 \leq x \leq 1$, $1 \leq y \leq 2$ and $3 \leq z \leq 4$.
32. $\iiint_Q (x + y + z)\, dV$, where Q is the tetrahedron bounded by $x + 2y + z = 4$ and the coordinate planes.
33. $\iiint_Q (x^2 + y^2)\, dV$, where Q is bounded by $z = 4 - x^2 - y^2$ and the xy-plane.
34. $\iiint_Q e^{x^2+y^2} dV$, where Q is bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 2$.
35. $\iiint_Q \sqrt{x^2 + y^2 + z^2}\, dV$, where Q is bounded by $z = \sqrt{x^2 + y^2}$ and $z = \sqrt{2 - x^2 - y^2}$.
36. $\iiint_Q (x^2 + y^2 + z^2)^{3/2}\, dV$, where Q is the solid below $z = -\sqrt{x^2 + y^2}$ and inside $z = -\sqrt{4 - x^2 - y^2}$.

In exercises 37–48, use an appropriate coordinate system to find the volume of the given solid.

37. The solid below $x^2 + y^2 + z^2 = 4z$ and above $z = \sqrt{x^2 + y^2}$
38. The solid above $z = \sqrt{x^2 + y^2}$ and below $x^2 + y^2 + z^2 = 4$
39. The solid inside $z = \sqrt{2x^2 + 2y^2}$ and between $z = 2$ and $z = 4$
40. The solid bounded by $z = 4x^2 + 4y^2$, $z = 0$, $x^2 + y^2 = 1$ and $x^2 + y^2 = 2$
41. The solid under $z = \sqrt{x^2 + y^2}$ and above the square $-1 \leq x \leq 1, -1 \leq y \leq 1$
42. The solid bounded by $x + 2y + z = 4$ and the coordinate planes
43. The solid below $x^2 + y^2 + z^2 = 4$, above $z = \sqrt{x^2 + y^2}$ in the first octant
44. The solid below $x^2 + y^2 + z^2 = 4$, above $z = \sqrt{x^2 + y^2}$, between $y = x$ and $x = 0$ with $y \geq 0$
45. The solid below $z = \sqrt{x^2 + y^2}$, above the xy-plane and inside $x^2 + y^2 = 4$
46. The solid between $z = 4 - x^2 - y^2$ and the xy-plane