MAT251 Multivariable Calculus III
Prof. PORTER

Name________________ Practice Test #1 FALL ~~MM~~ 2014

1. What is Multivariable Calculus?

Study of change in multiple dimensions - PA

Why would your major want you to know this material?

My major is about change / and life is 3D.

Given the points A(3,1,2), B(-2,0,1), and C(0,2,0), Find the angle between the vectors **BC** and **AC**

$\vec{BC} = \langle 2,2,-1 \rangle \quad \vec{AC} = \langle -3,1,-2 \rangle$

$\cos\theta = \frac{\vec{BC}\cdot\vec{AC}}{\|\vec{BC}\|\|\vec{AC}\|} = \frac{-2}{3\sqrt{14}} \therefore \theta = 100.26°$

Answer ZH

2. Given the points A(3,1,2), B(-2,0,1), and C(0,2,0), Find the parametric equation of the line containing **AB**

$\vec{AB} = \langle -5,-1,-1 \rangle$

$= A = (3,1,2)$

$x(t) = 3-5t$
$y(t) = 1-t$
$z(t) = 2-t$

(VJ)

Answer____________

Does **AB** cross s(t) = < -14s, 2 - 6s, 2 - 4s > ?

$3-5t = -14s$
$-5(1-t = 2-6s)$
$-2+0 = -10+16s$
$s = \frac{1}{2}$
$-7 = 3-5t$
$t = 2$

Answer Yes

S.T.

cross
$(-7,-1,0)$

3. Given the points A(3,1,2), B(-2,0,1), and C(0,2,0):

a) Find the equation of the plane containing A, B, and C

$\vec{AB} = \langle -2-3,\ 0-1,\ 1-2 \rangle$

$= \langle -5,\ -1,\ -1 \rangle$

$\vec{AC} = \langle 0-3,\ 2-1,\ 0-2 \rangle$

$= \langle -3,\ 1,\ -2 \rangle$

$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -5 & -1 & -1 \\ -3 & 1 & -2 \end{vmatrix}$

$= \begin{vmatrix} -1 & -1 \\ 1 & -2 \end{vmatrix} i - \begin{vmatrix} -5 & -1 \\ -3 & -2 \end{vmatrix} j + \begin{vmatrix} -5 & -1 \\ -3 & 1 \end{vmatrix} k$

$= \langle 3,\ -7,\ -8 \rangle$

Answer $3x - 7y - 8z = -14$

$(0,2,0)$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$3(x-0) + (-7)(y-2) + (-8)(z-0) = 0$

$3x + (-7y) + 14 - 8z = 0$

$3x - 7y - 8z = -14$

Peter

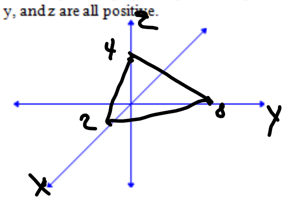
b) Given 8x + 2y + 4z = 16 Find the intercepts and graph the plane. Graph the intercepts and the plane in the first octant. Draw a graph of the plane when x, y, and z are all positive.

DC

$y=0,\ z=0$: $8x = 16$, $x = 2$

$x=0,\ y=0$: $4z = 16$, $z = 4$

$x=0,\ z=0$: $2y = 16$, $y = 8$



4. Given the points A(3,1,2), B(-2,0,1), and C(0,2,0):

a) Is the point (1,-2,9) on the plane containing A, B, and C? Why?

Answer No

Xiao

b) What is the distance between (1,3,0) and the plane containing A, B, and C

Answer____________

a) $-14 = 3x - 7y - 8z$

$(1,-2,9)$

$3 + 14 - 72$

$= 55 \neq -14$

$$D = \frac{|A(x_1) + B(x_2) + C(x_3) + D|}{\sqrt{A^2 + B^2 + C^2}}$$

$$= \frac{|3(1) - 7(3) - 8(0) + 14|}{\sqrt{(3)^2 + (-7)^2 + (-8)^2}}$$

$$= \frac{4}{\sqrt{122}} = 0.36 \text{ units}$$

SR

- $\frac{x^2}{16} - \frac{y^2}{9} - \frac{z^2}{4} = 1$

$x=0 \quad -\frac{y^2}{9} - \frac{z^2}{4} = 1$

$y=0$ HYPERBOLA

$z=0$ HYPERBOLA

HYPERBOLOID 2 SHEETS

DOBO

- $\frac{x^2}{16} - \frac{y}{9} - \frac{z^2}{4} = 1$

Collin

$x=0 \quad -y-z^2=C$ parabola down

$y=0 \quad x^2-z^2=C$ hyperbola

$z=0 \quad x^2-y=C$ parabola up

hyperbolic parabola

- $\frac{x^2}{16} - \frac{y^2}{9} - \frac{(z-2)^2}{4} = 0$

Alex

Cone

$x=0 \quad -y^2-(z-2)^2=0$

$-y^2=(z-2)^2 \quad 0$

$y=0 \quad z=2$

$y=0 \quad x^2-(z-2)^2=0$

$x^2=(z-2)^2 \quad x=\pm z-2$

$z=0 \quad x^2-y^2=0 \to x=\pm y$

6. Draw the level curves on the x-y plane for z at 0, 1, 2 for the surface:

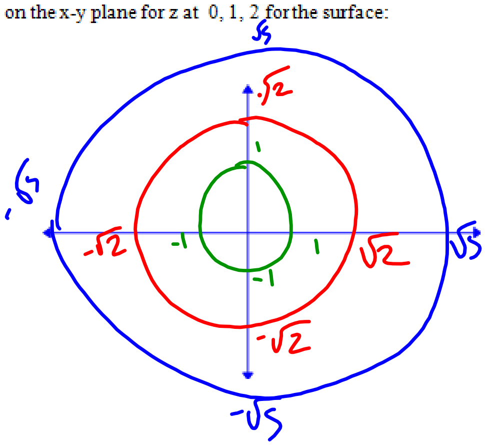
$x^2 + y^2 - z^2 = 1$

$z = 0$
$x^2 + y^2 = 1$

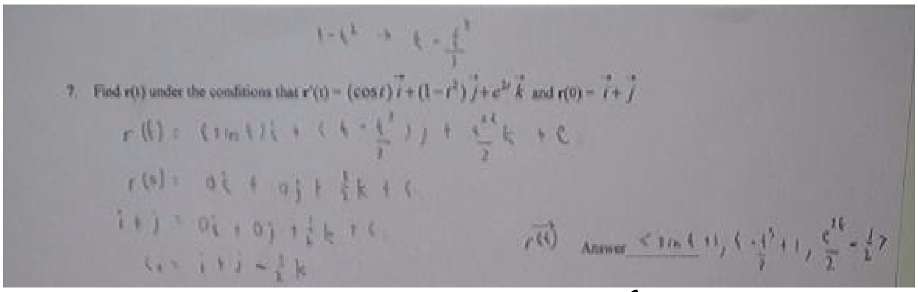
$z = 1$
$x^2 + y^2 = 2$

$z = 2$
$x^2 + y^2 = 5$

Deep



7. Find **r**(t) under the conditions that **r**'(t) = $(\cos t)\vec{i} + (1-t^2)\vec{j} + e^{2t}\vec{k}$ and **r**(0) = $\vec{i} + \vec{j}$



$1 - t^2 \to t - \frac{t^3}{3}$

7. Find r(t) under the conditions that r'(t) = $(\cos t)\vec{i} + (1-t^2)\vec{j} + e^{2t}\vec{k}$ and r(0) = $\vec{i} + \vec{j}$

$r(t) = (\sin t)i + (t - \frac{t^3}{3})j + \frac{e^{2t}}{2}k + C$

$r(0) = 0i + 0j + \frac{1}{2}k + C$

$i + j = 0i + 0j + \frac{1}{2}k + C$

$C = i + j - \frac{1}{2}k$

$r(t)$ Answer $\langle \sin t + 1, t - \frac{t^3}{3} + 1, \frac{e^{2t}}{2} - \frac{1}{2} \rangle$

$$\left\langle \sin t + 1,\ t - \frac{t^3}{3} + 1,\ \frac{e^{2t}}{2} - \frac{1}{2} \right\rangle$$

Benito

8. Given **r**(t) = <2t+3,0,t²>, find the distance between **r**(3) and **r**(0).

$r(3) = \langle 2(3)+3, 0, (3)^2 \rangle$

$r(3) = \langle 9, 0, 9 \rangle$

$r(0) = \langle 2(0)+3, 0, (0)^2 \rangle$

$r(0) = \langle 3, 0, 0 \rangle$

Answer $\sqrt{117}$

J.B

$r(3) - r(0)$

$\langle 9,0,9 \rangle - \langle 3,0,0 \rangle = \langle 6,0,9 \rangle$

$\sqrt{(6)^2+(0)^2+(9)^2} = \sqrt{36+0+81} = \sqrt{117}$

What is the length of the curve **r**(t) = <2t+3,0,t²> over the interval [0,3]?

$S = \int_a^b \sqrt{(x')^2+(y')^2+(z')^2}\,dt$

$S = \int_0^3 \sqrt{(2)^2 + 0 + (2t)^2}\,dt$

Answer

Definite integral:

$$\int_0^3 \sqrt{4+4t^2}\,dt = 3\sqrt{10} + \sinh^{-1}(3) \approx 11.305$$

Jon

$Y_1 = \sqrt{(4+4x^2)}$

Calc 7: $\int f(x)dx$

Lower: 0 Upper: 3

$\int f(x)dx = 11.305$

fnint $(\sqrt{(4+4x^2)}, x, 0, 3)$

. Given the position vector $r(t) = \langle 5t, 2\sin t, 2\cos t\rangle$

a) Find the acceleration at $t = 0$

$r'(t) = \langle 5, 2\cos t, -2\sin t\rangle$

$r''(t) = \langle 0, -2\sin t, -2\cos t\rangle$

$r''(0) = \langle 0, 0, -2\rangle$

JD

Answer___________

b) Find the speed at $t = 0$

$R'(T) = (5, 2\cos T, -2\sin T)$

$R'(0) = (5, 2, 0)$

$\sqrt{5^2 + 2^2 + 0^2} =$

Answer $\sqrt{29}$

0.

Give the equation of the line of intersection between the plane $x + y = 1$ and the plane $2x + 3z = 6$?

$x = 1 - y$

$2(1-y) + 3z = 6$

$2 - 2y + 3z = 6$

$3z = 4 + 2y$

$z = \frac{4+2y}{3}$

$x = t$

$y = 1 - t$

$z = 2 - \frac{2}{3}t$

$2t + 3z = 6$

$\frac{6 - 2t = 3z}{3}$

ok

10.

Give the equation of the line of intersection between the plane $x + y = 1$ and the plane $2x + 3z = 6$?

$A \Rightarrow x + y + 0z = 1$

$B \Rightarrow 2x + 0y + 3z = 6$

$\vec{N}_A = \langle 1, 1, 0 \rangle$

$\vec{N}_B = \langle 2, 0, 3 \rangle$

Answer________

point in plane A

$(0, 1, 2)$

$$\vec{N}_A \times \vec{N}_B = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 3\hat{i} - 3\hat{j} - 2\hat{k} = \langle 3, -3, -2 \rangle$$

$$\boxed{\begin{aligned} x(t) &= 1 + 3t \\ y(t) &= -3t + 1 \\ z(t) &= -2t + 2 \end{aligned}}$$

9. Given the position vector $r(t) = \langle 5t, 2\sin t, 2\cos t\rangle$

a) Find the acceleration at $t = 0$

$r'(t) = \langle 5, 2\cos t, -2\sin t\rangle$ $r''(0) = \langle 0, 0, -2\rangle$

$r''(t) = \langle 0, -2\sin t, -2\cos t\rangle$

Answer____________

b) Find the speed at $t = 0$

Answer____________

10. Give the equation of the line of intersection between the plane $x + y = 1$ and the plane $2x + 3z = 6$?