Find the tangential and normal components of acceleration for an object with position vector $\mathbf{r}(t) = \langle 3 \sin t, 3 \cos t, 7t \rangle$. Use $\kappa = \frac{3}{58}$.

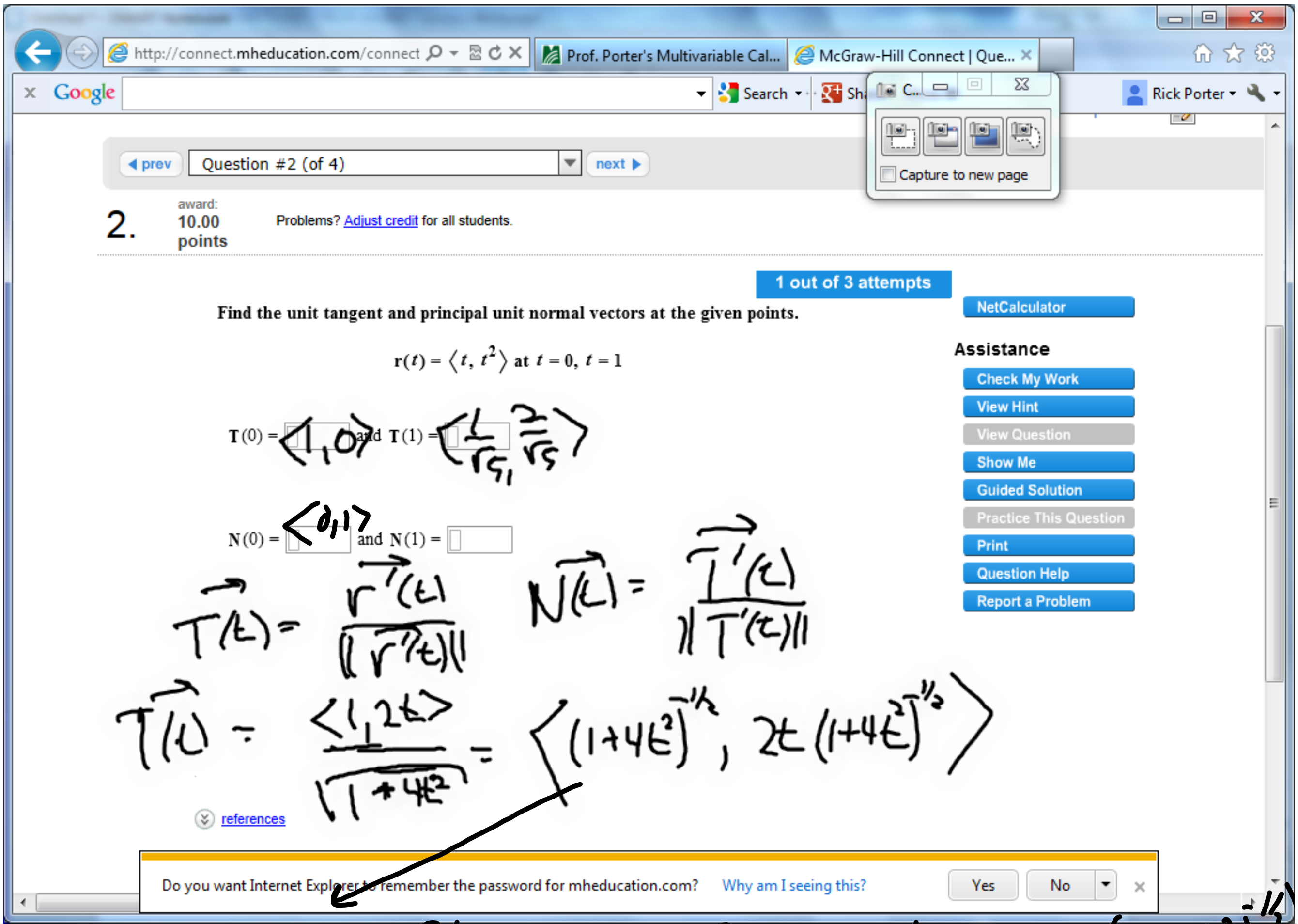
We have $\mathbf{r}'(t) = \langle 3 \cos t, -3 \sin t, 7 \rangle$, so that

$$\frac{ds}{dt} = ||\mathbf{r}'(t)|| = \sqrt{58}$$

and so, $\frac{d^2s}{dt^2} = 0$, for all t. We have that the acceleration is

$$\begin{aligned} \mathbf{a}(t) &= \frac{d^2s}{dt^2}\mathbf{T}(t) + \kappa\left(\frac{ds}{dt}\right)^2 \mathbf{N}(t) \\ &= (0)\mathbf{T}(t) + \frac{3}{58}\left(\sqrt{58}\right)^2 \mathbf{N}(t) = 3\mathbf{N}(t). \end{aligned}$$

So, here we have $\mathbf{a_T} = 0$ and $\mathbf{a_N} = 3$.



$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|} \qquad \vec{N}(t) = \frac{\vec{T}\,'(t)}{\|\vec{T}\,'(t)\|}$$

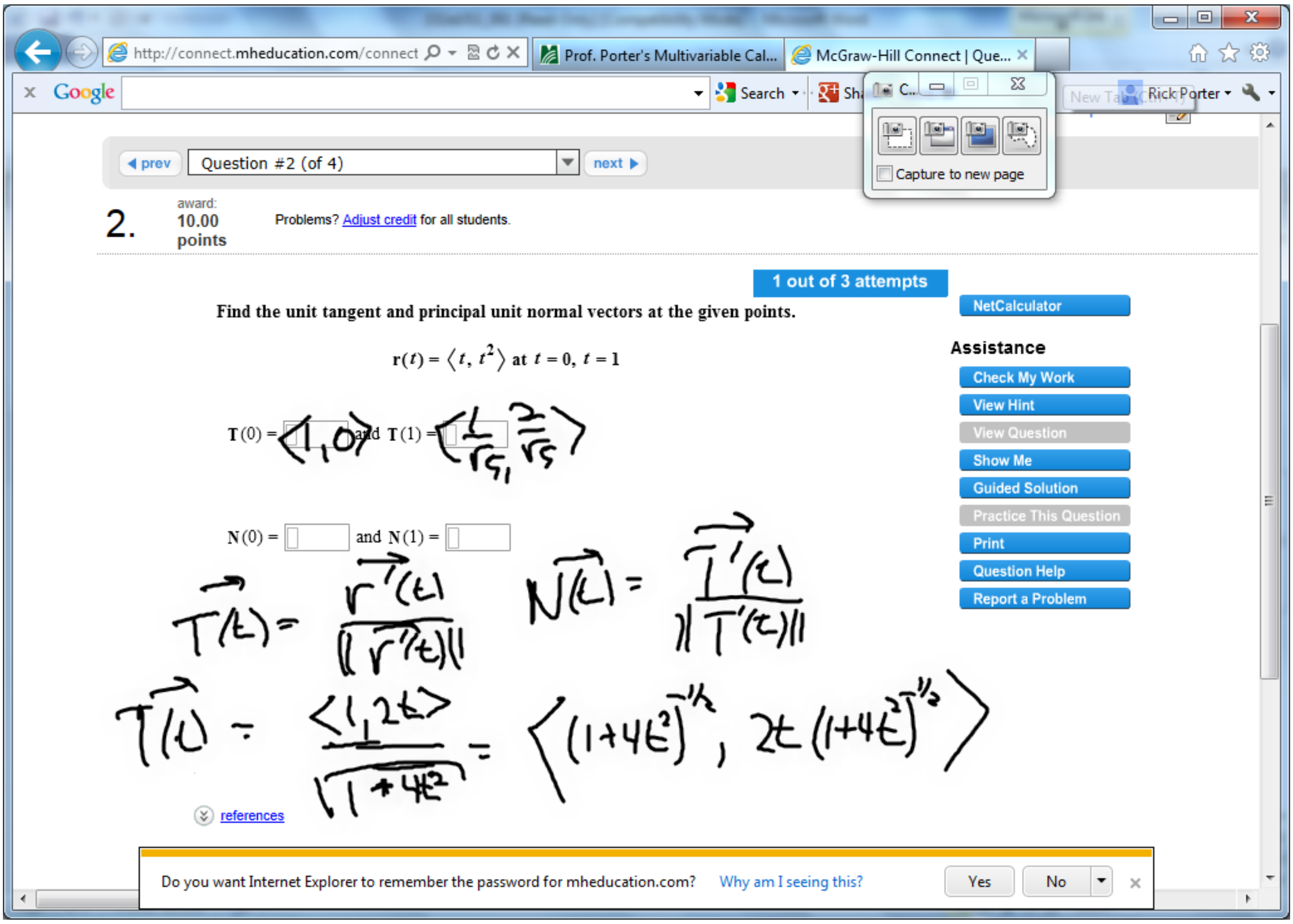
$$\vec{T}(t) = \frac{\langle 1, 2t\rangle}{\sqrt{1+4t^2}} = \left\langle (1+4t^2)^{-1/2},\ 2t(1+4t^2)^{-1/2} \right\rangle$$

$$\vec{T}\,'(t) = \left\langle -\tfrac{1}{2}(1+4t^2)^{-3/2}\cdot(8t),\ 2t\left[-\tfrac{1}{2}(1+4t^2)^{-3/2}(8t)\right] + 2(1+4t^2)^{-1/2} \right\rangle$$

$$\frac{\vec{T}\,'(0)}{\|T'(0)\|} = \frac{\langle 0,2\rangle}{2} = \langle 0,1\rangle$$

$$T'(1) = \left\langle \frac{-4}{\sqrt{125}},\ \# \right\rangle$$

→ make unit


http://connect.mheducation.com/connect
Prof. Porter's Multivariable Cal...
McGraw-Hill Connect | Que...
Google
Search
Capture to new page
Rick Porter
prev
Question #2 (of 4)
next
2.
award:
10.00
points
Problems? Adjust credit for all students.
1 out of 3 attempts
Find the unit tangent and principal unit normal vectors at the given points.
r(t) = ⟨t, t²⟩ at t = 0, t = 1
T(0) = ⟨1, 0⟩ and T(1) = ⟨1/√5, 2/√5⟩
N(0) = and N(1) =
T(t) = r′(t) / ‖r′(t)‖
N(t) = T′(t) / ‖T′(t)‖
T(t) = ⟨1, 2t⟩ / √(1 + 4t²) = ⟨(1+4t²)^(-1/2), 2t (1+4t²)^(-1/2)⟩
references
NetCalculator
Assistance
Check My Work
View Hint
View Question
Show Me
Guided Solution
Practice This Question
Print
Question Help
Report a Problem
Do you want Internet Explorer to remember the password for mheducation.com?
Why am I seeing this?
Yes
No

$\mathbf{r}'(t) = \left\langle 1, 2t \right\rangle$ and $\|\mathbf{r}'(t)\| = \sqrt{1 + 4t^2}$,

so we have $\mathbf{T}(t) = \left\langle \dfrac{1}{\sqrt{1+4t^2}}, \dfrac{2t}{\sqrt{1+4t^2}} \right\rangle$.

This yields $\mathbf{T}(0) = \left\langle 1, 0 \right\rangle$ and $\mathbf{T}(1) = \left\langle \dfrac{1}{\sqrt{5}}, \dfrac{2}{\sqrt{5}} \right\rangle$.

Also, since $\mathbf{T}'(t) = \left\langle \dfrac{-4t}{\left(1+4t^2\right)^{3/2}}, \dfrac{2}{\left(1+4t^2\right)^{3/2}} \right\rangle$ and $\|\mathbf{T}'(t)\| = \dfrac{2}{1+4t^2}$, we have

$\mathbf{N}(t) = \left\langle \dfrac{-2t}{\sqrt{1+4t^2}}, \dfrac{1}{\sqrt{1+4t^2}} \right\rangle$.

This yields $\mathbf{N}(0) = \left\langle 0, 1 \right\rangle$ and $\mathbf{N}(1) = \left\langle \dfrac{-2}{\sqrt{5}}, \dfrac{1}{\sqrt{5}} \right\rangle$.

Find the tangential and normal components of acceleration for the given position function at the given points.

$$\mathbf{r}(t) = \left\langle 8t,\ 16t - 16t^2 \right\rangle \text{ at } t = 0,\ t = 1$$

$\mathbf{v}(t) = \left\langle 8,\ 16 - 32t \right\rangle$ and $\mathbf{a}(t) = \left\langle 0,\ -32 \right\rangle$.

$$\|\mathbf{v}(t)\| = \frac{ds}{dt} = 8\sqrt{5 - 16t + 16t^2},$$

so the tangential component is

$$a_T = \frac{d^2s}{dt^2} = \frac{64(2t - 1)}{\sqrt{5 - 16t + 16t^2}}.$$

The normal component is

$$a_N = \sqrt{\|\mathbf{a}(t)\|^2 - a_T{}^2} = \frac{32}{\sqrt{5 - 16t + 16t^2}}.$$

At $t = 0$, $a_T = \frac{-64}{\sqrt{5}}$ and $a_N = \frac{32}{\sqrt{5}}$.

The friction force required to keep a car from skidding on a curve is given by $\mathbf{F}_s(t) = m a_N \mathbf{N}(t)$. Select the friction force needed to keep a car of mass $m = 100$ (slugs) from skidding.

$\mathbf{r}(t) = \langle 300 \cos 2\pi t, 300 \sin 2\pi t \rangle$

Since $\|\mathbf{v}(t)\| = 600\pi$, $\|\mathbf{a}(t)\| = 1200\pi^2$, and $a_T = \frac{d}{dt}\|\mathbf{v}(t)\| = 0$, we know

$$
\begin{aligned}
a_N &= \sqrt{\|\mathbf{a}(t)\|^2 - a_T{}^2} = 1200\pi^2. \\
\text{Thus, } \mathbf{F}_s(t) &= m a_N \mathbf{N}(t) \\
&= 120000\pi^2 \langle -\cos 2\pi t, -\sin 2\pi t \rangle.
\end{aligned}
$$

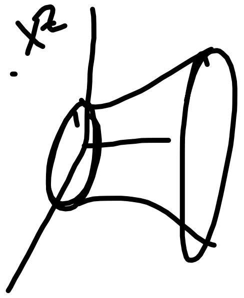
Identify the surface defined by the parametrically by $x = 2 \sin u \cosh v$, $y = 2 \sinh v$, $z = 2 \cos u \cosh v$, $0 \le u \le 2\pi$ and $-\infty < v < \infty$.

- A. The surface is a sphere centered at the origin with radius $\sqrt{2}$.
- B. The surface is a hyperboloid of one sheet wrapped around the $z-$axis.
- C. The surface is a hyperboloid of one sheet wrapped around the $y-$axis.
- D. The surface is a sphere centered at the origin with radius 2.

$x^2 = 4\sin^2 u \cosh^2 v$

$z^2 = 4\cos^2 u \cosh^2 v$

$x^2+z^2 = 4\cosh^2 v(\sin^2 u + \cos^2 u)$



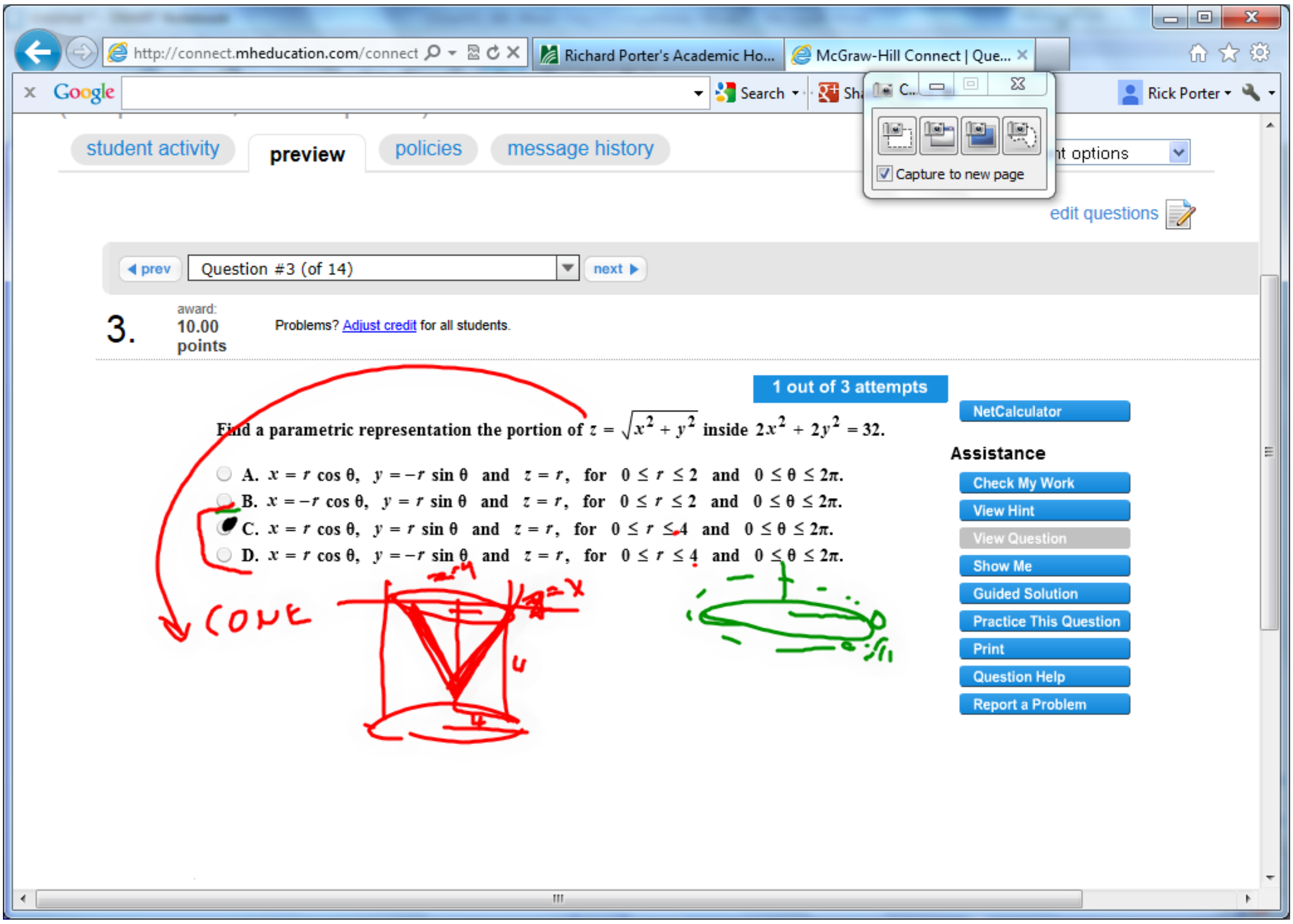
$x^2+z^2 = 4\cosh^2 v$

$y^2 = 4\sinh^2 v$

$x^2+z^2-y^2 = 4(\cosh^2 v - \sinh^2 v)$

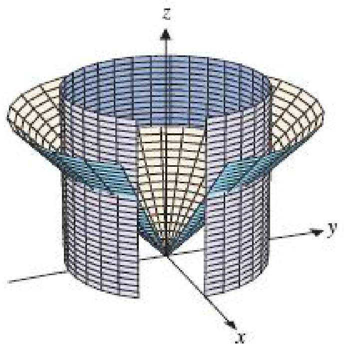
$x^2+z^2-y^2 = 4$

Hyp. of 1 Sheet


student activity
preview
policies
message history
edit questions
Question #3 (of 14)
3.
award: 10.00 points
Problems? Adjust credit for all students.
1 out of 3 attempts
Find a parametric representation the portion of $z = \sqrt{x^2 + y^2}$ inside $2x^2 + 2y^2 = 32$.
A. $x = r\cos\theta$, $y = -r\sin\theta$ and $z = r$, for $0 \le r \le 2$ and $0 \le \theta \le 2\pi$.
B. $x = -r\cos\theta$, $y = r\sin\theta$ and $z = r$, for $0 \le r \le 2$ and $0 \le \theta \le 2\pi$.
C. $x = r\cos\theta$, $y = r\sin\theta$ and $z = r$, for $0 \le r \le 4$ and $0 \le \theta \le 2\pi$.
D. $x = r\cos\theta$, $y = -r\sin\theta$ and $z = r$, for $0 \le r \le 4$ and $0 \le \theta \le 2\pi$.
CONE
NetCalculator
Assistance
Check My Work
View Hint
View Question
Show Me
Guided Solution
Practice This Question
Print
Question Help
Report a Problem

Show Me

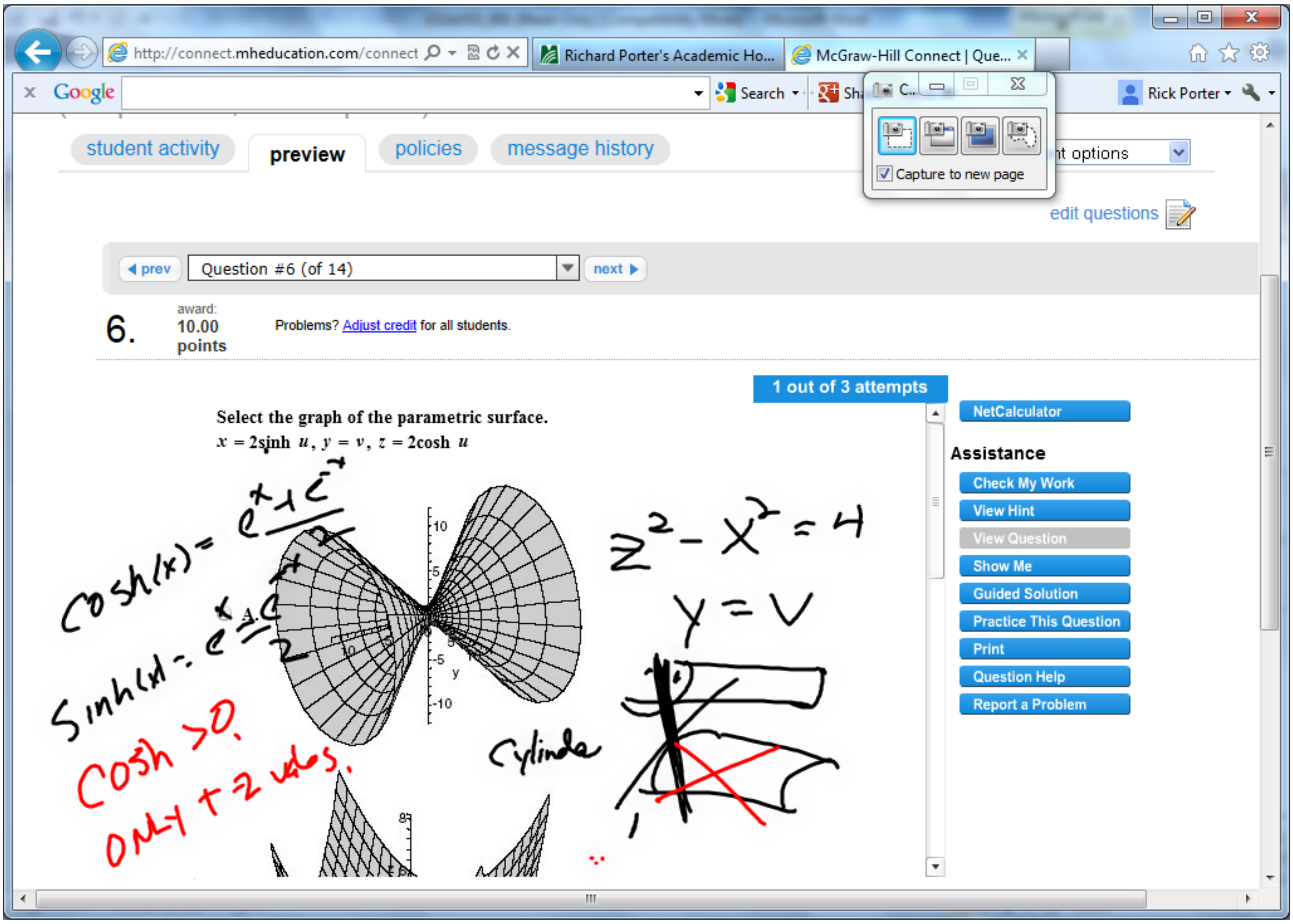
A graph indicating the cone and the cylinder is shown in below.



Dividing both sides of the equation $2x^2 + 2y^2 = 32$ by 2 yields $x^2 + y^2 = 16$.

Notice that the equations for both surfaces include the term $x^2 + y^2$ and x and y appear only in this combination. This suggests the use of polar coordinates.

Taking $x = r\cos\theta$ and $y = r\sin\theta$, the equation of the cone $z = \sqrt{x^2 + y^2}$ becomes $z = r$ and the equation of the cylinder $x^2 + y^2 = 16$ becomes $r = 4$.


student activity
preview
policies
message history
edit questions
Question #6 (of 14)
6.
award:
10.00
points
Problems? Adjust credit for all students.
1 out of 3 attempts
Select the graph of the parametric surface.
x = 2sinh u, y = v, z = 2cosh u
NetCalculator
Assistance
Check My Work
View Hint
View Question
Show Me
Guided Solution
Practice This Question
Print
Question Help
Report a Problem

7

Select the graph of the parametric surface.

$x = v\sinh u$, $y = 4v^2$, $z = v\cosh u$

$x = v\sinh u$, $y = 4v^2$, $z = v\cosh u$

$x^2 = v^2\sinh^2 u$, $y^2 = 16v^4$, $z^2 = v^2\cosh^2 u$

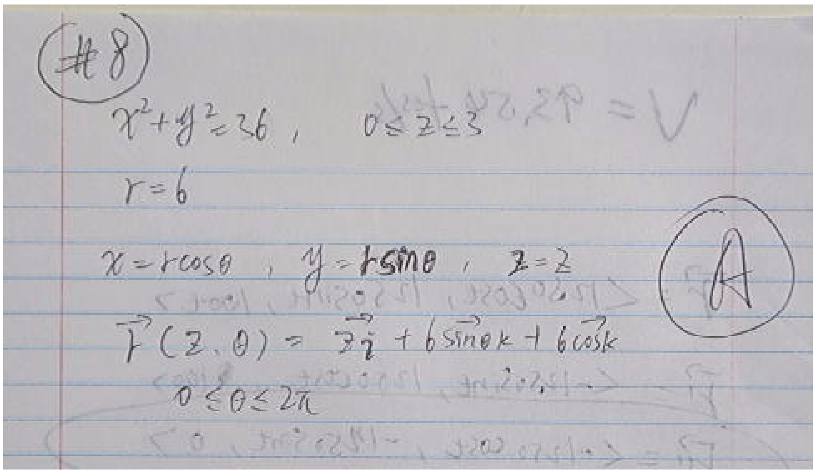
$z^2 - x^2 = y$

$\therefore$

Hyperbolic on xz plane

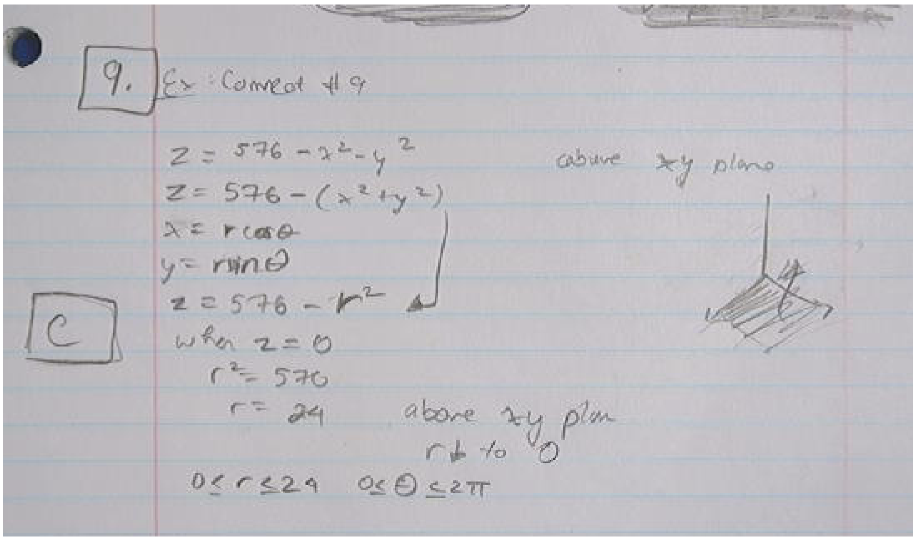
Select a parametric representation of the portion of $x^2 + y^2 = 36$ from $z = 0$ to $z = 3$.

- **A.** $x = 6\cos\theta$ and $y = 6\sin\theta$ for $0 \le \theta \le 2\pi$ and $0 \le z \le 3$.
- **B.** $x = 6\cos\theta$ and $y = -6\sin\theta$ for $0 \le \theta \le 2\pi$ and $0 \le z \le 3$.
- **C.** $x = 6\cos\theta$ and $y = 6\sin\theta$ for $0 \le \theta \le 2\pi$ and $0 \le z \le 6$.
- **D.** $x = 6\cos\theta$ and $y = -6\sin\theta$ for $0 \le \theta \le 2\pi$ and $0 \le z \le 6$.



Find a parametric representation of the portion of $z = 576 - x^2 - y^2$ above the xy–plane.

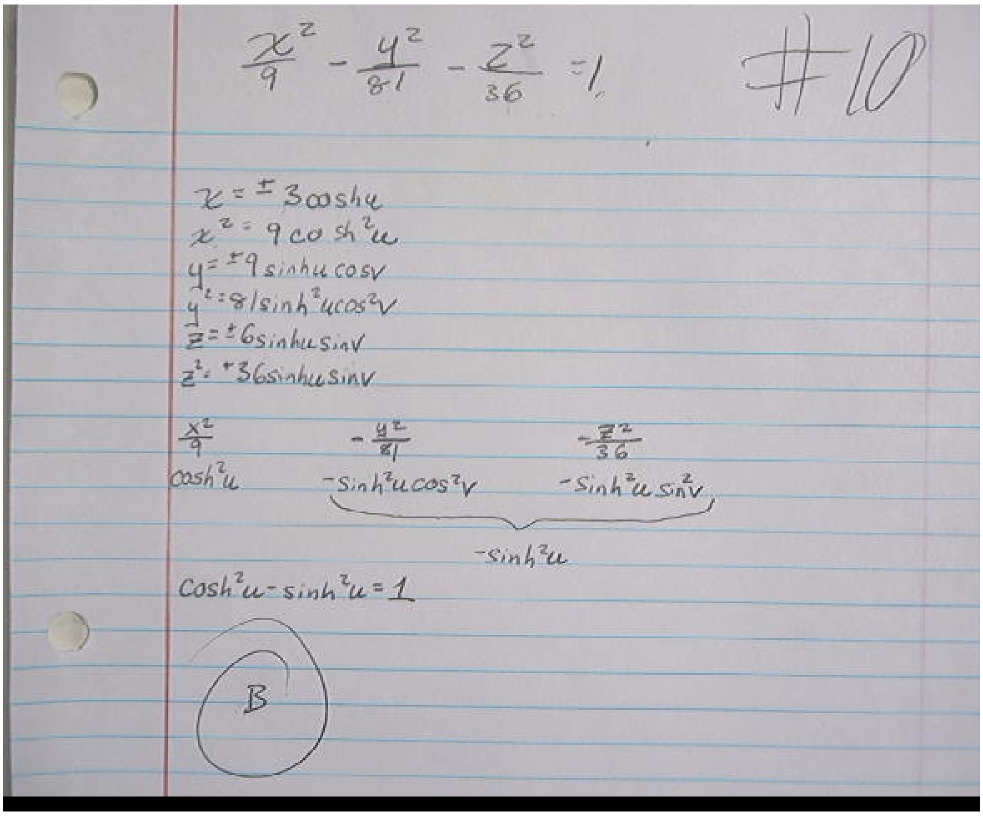
- A. $x = r\cos\theta$, $y = r\sin\theta$, and $z = 24 - r^2$ for $0 \le r \le 24\pi$ and $0 \le \theta \le 2\pi$.
- B. $x = r\cos\theta$, $y = r\sin\theta$, and $z = -576 + r^2$ for $0 \le r \le 24\pi$ and $0 \le \theta \le 2\pi$.
- C. $x = r\cos\theta$, $y = r\sin\theta$, and $z = 576 - r^2$ for $0 \le r \le 24$ and $0 \le \theta \le 2\pi$.
- D. $x = r\cos\theta$, $y = r\sin\theta$, and $z = -24 + r^2$ for $0 \le r \le 24$ and $0 \le \theta \le 2\pi$.



Select the parametric representation of the surface.

$$\frac{x^2}{9} - \frac{y^2}{81} - \frac{z^2}{36} = 1$$

- A. $x = \pm\cosh u$, $y = 9 \sinh u$, and $z = 3 \sinh u \cos v$ where $-\infty \le u \le \infty$ and $0 \le v \le 2\pi$.
- B. $x = \pm 3 \cosh u$, $y = 9 \sinh u \cos v$, and $z = 6 \sinh u \sin v$ where $-\infty \le u \le \infty$ and $0 \le v \le 2\pi$.
- C. $x = \pm\cosh u$, $y = 9 \sinh u$, and $z = 6 \sinh u \sin v$ where $-\infty \le u \le \infty$ and $0 \le v \le 2\pi$.
- D. $x = \pm 3 \cosh u$, $y = 9 \sinh u \cos v$, and $z = 3 \sinh u \cos v$ where $-\infty \le u \le \infty$ and $0 \le v \le 2\pi$.



$\frac{x^2}{9} - \frac{y^2}{81} - \frac{z^2}{36} = 1$ #10

$x = \pm 3\cosh u$

$x^2 = 9\cosh^2 u$

$y = \pm 9\sinh u \cos v$

$y^2 = 81\sinh^2 u \cos^2 v$

$z = \pm 6\sinh u \sin v$

$z^2 = +36 \sinh u \sin v$

$\frac{x^2}{9}$ $\quad -\frac{y^2}{81}$ $\quad -\frac{z^2}{36}$

$\cosh^2 u$ $\quad -\sinh^2 u \cos^2 v$ $\quad -\sinh^2 u \sin^2 v$

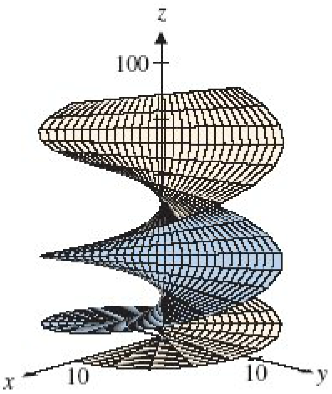
$-\sinh^2 u$

$\cosh^2 u - \sinh^2 u = 1$

B

Select the parametric equation for the surface.

11



- A. $x = v$, $y = u \cos v$, $z = u \sin v$
- B. $x = u \cos v$, $y = u \sin v$, $z = v^2$
- C. $x = u$, $y = u \cos v$, $z = u \sin v$
- D. $x = u$, $y = \sin u \cos v$, $z = \sin u \sin v$

Relate to cylindrical coordinates defined by $x = r\cos\theta$, $y = r\sin\theta$ and $z = z$.

Select parametric equations for the wedge in the first octant bounded by $y = 0$, $y =$ $x^2 + y^2 = 196$, $z = 0$ and $z = 8$.

- A. $x = r\cos\theta$, $y = r\sin\theta$, $z = z$ gives the desired region with $0 \le \theta \le \frac{\pi}{4}$,
  $0 \le r \le 14$ and
  $0 \le z \le 8$.
- B. $x = r\cos\theta$, $y = r\sin\theta$, $z = z$ gives the desired region with $0 \le \theta \le \frac{\pi}{3}$,
  $0 \le r \le 196$ and
  $0 \le z \le 8$.
- C. $x = r\cos\theta$, $y = r\sin\theta$, $z = z$ gives the desired region with $0 \le \theta \le \frac{\pi}{4}$,
  $0 \le r \le 8$ and
  $0 \le r \le 14$.
- D. $x = r\cos\theta$, $y = r\sin\theta$, $z = z$ gives the desired region with $0 \le \theta \le \frac{\pi}{4}$,

13

$x^2 + y^2 = 196$ $z = 0$ $z = 8$

$\sqrt{196} = 14$

$x = r\cos\theta$, $y = r\sin\theta$, $z = z$ gives the desired region

$0 \leq r \leq 14$ with $0 \leq \theta \leq \frac{\pi}{4}$

$0 \leq z \leq 8$