Find the area of the triangle with vertices a = (3, 5, 4), b = (2, 4, 1), and c = (0, 0, 0). Round your final answer to three decimal places.

The area of the triangle is ______ .

$\langle 3, 5, 4 \rangle$

$\langle 2, 4, 1 \rangle$

$$A_T = \frac{\| \langle 3,5,4 \rangle \times \langle 2,4,1 \rangle \|}{2}$$

$$V = \begin{vmatrix} i & j & k \\ 3 & 5 & 4 \\ 2 & 4 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} 5 & 4 \\ 4 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix}$$

$$= -11i + 5 + 2k$$

$$\|V\| = \sqrt{121+25+4} = \frac{\sqrt{150}}{2} = 6.12$$

Find an equation of the line through the point (3, 2, 3) and parallel to the vector $\langle 3, 4, 6 \rangle$. Also, determine where the line intersects the yz–plane.

A. $L = \frac{x-3}{3} = \frac{y-2}{4} = \frac{z-3}{6}$; (0, −2, −3)

B. $\frac{x-4}{7} = \frac{y+6}{4} = \frac{z-3}{6}$; (1, −2, −3)

C. $\frac{x+3}{3} = \frac{y+2}{4} = \frac{z+3}{6}$; (0, 1, 12)

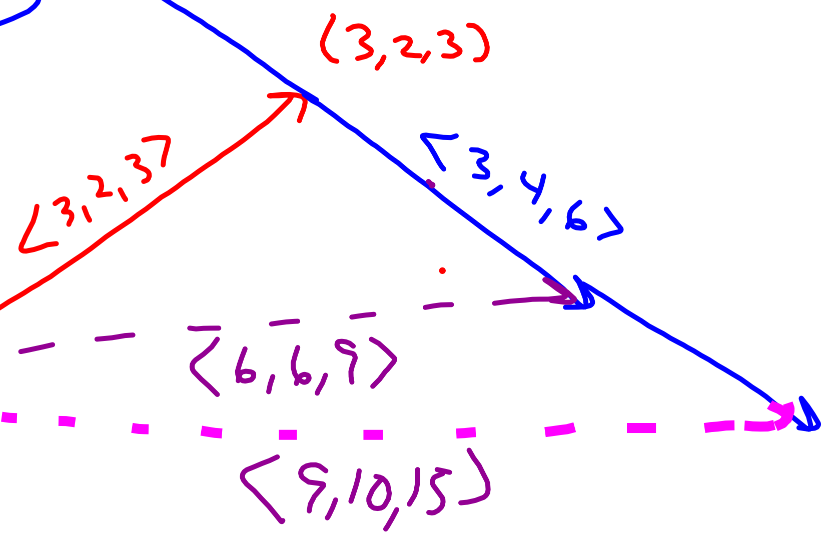
D. $\frac{x-4}{7} = \frac{y+2}{4} = \frac{z+3}{6}$; (0, −2, −3)

$x = 3 + 3t = 0$

$y = 2 + 4t$

$z = 3 + 6t$

$t = -1$



Determine if the lines

ℓ_1: $x-7=-t$, $y-7=2t$, and $z-2=2t$

ℓ_2: $x-6=s$, $y-8=-s$, and $z-2=3s$

are parallel or intersect.

$\langle x,y,z\rangle = \langle 7,7,2\rangle + t\langle -1,2,2\rangle$

$\langle 6,8,2\rangle + s\langle 1,-1,3\rangle$

$-1 = -t - s$

$1 = 2t + s$

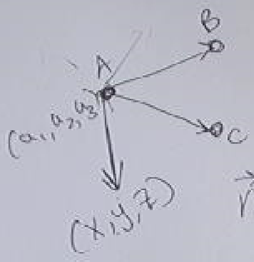
$0 = t$

$1 = s$

$2(0) \overset{?}{=} 3(1)$

$\#\langle -1,2,2\rangle \neq \langle 1,-1,3\rangle$

Equation of a Plane



$\vec{AB} \times \vec{AC} = \vec{n}$

normal to plane

$\vec{n} \cdot \langle x-a_1, y-a_2, z-a_3 \rangle = 0$

$0 = \langle n_1, n_2, n_3 \rangle \cdot \langle x-a_1, y-a_2, z-a_3 \rangle$

$0 = n_1(x-a_1) + n_2(y-a_2) + n_3(z-a_3)$

$0 = n_1 x + n_2 y + n_3 z + \#$

Ex $3x + 2y + 4z = 8$

normal $= \langle 3, 2, 4 \rangle$

Supporting Work:

A. head $(2, 1, 1.5)$
B. Chair $(4, 2, 3)$
C. Door $(1, 6, 1)$

$\vec{AB} \times \vec{AC} = \vec{m}$
$\vec{AB} = \langle 2, 1, 1.5 \rangle$
$\vec{AC} = \langle -1, 5, -0.5 \rangle$

$\langle 2, 1, 1.5 \rangle \times \langle -1, 5, -0.5 \rangle = \vec{m}$

$\vec{m} = \langle -8, 0.5, 11 \rangle$

$\langle -8, 0.5, 11 \rangle \cdot \langle x - a_1, y - a_2, z - a_3 \rangle = 0$

$-8(x - a_1) + 0.5(y - a_2) + 11(z - a_3) = 0$

$\downarrow$ 2 $\quad \downarrow$ 1 $\quad \downarrow$ 1.5

$-8x + 8a_1 + 0.5y - 0.5a_2 + 11z - 11a_3 = 0$
$-8x + 8(2) + 0.5y - 0.5(1) + 11z - 11(1.5) = 0$
$-8x + 16 + 0.5y - 0.5 + 11z - 16.5 = 0$
$-8x + 0.5y + 11z - 1 = 0$
$-8x + 0.5y + 11z = 1$

Conclusion (in words):

Prof. Porter isn't in danger

Supporting Work:

1. H: $\langle 2,4,1 \rangle$ $\quad \vec{GH} = \langle -2, 2, -3 \rangle$

D: $\langle 1,7,1 \rangle$ $\therefore$ $\vec{GD} = \langle -3, 5, -3 \rangle$

G: $\langle 4,2,4 \rangle$

2. $$\begin{vmatrix} i & j & k \\ 2 & -2 & 3 \\ -3 & 5 & 3 \end{vmatrix} = i \begin{vmatrix} -2 & 3 \\ 5 & 3 \end{vmatrix} - j \begin{vmatrix} 2 & 3 \\ -3 & 3 \end{vmatrix} + k \begin{vmatrix} 2 & -2 \\ -3 & 5 \end{vmatrix}$$

$$= 9i + 3j - 4k$$

$$\therefore V_n = \langle 9, 3 - 4 \rangle$$

3. plug in a point (Ghost):

$$9(x-2) + 3(y-4) - 4(z-1) = 0$$

4. Is Prof. Porter in danger? $\quad P_{porter} = (2,2,1)$

$$9(2-2) + 3(2-4) - 4(1-1) \neq 0 \quad \therefore \boxed{\text{NO}}$$

Normal $= \langle A, B, C \rangle = \vec{n}$

Given $(0,0,0)$ $(1,1,1)$ $(1,2,3)$

Find EQ of Plane.

$\vec{v}_1 \langle 1,1,1 \rangle$ $\vec{v}_2 \langle 1,2,3 \rangle$

$\vec{n} = \vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \langle 1, -2, 1 \rangle$

$1x - 2y + 1z = D$

$1(1) - 2(2) + 1(3) = 0$

$x - 2y + z = 0$

Distance between a point (x_0, y_0, z_0) & a plane $Ax + By + Cz + D =$

$$D = \frac{|A(x_0) + B(y_0) + C(z_0) + D|}{\sqrt{A^2 + B^2 + C^2}}$$

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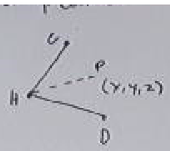
Supporting Work:

Ghost: $\langle 4,3,2\rangle$
Door: $\langle 1,6,1\rangle$
Head: $\langle 8,3,1\rangle$



$\overrightarrow{HG} = \langle 4,0,-1\rangle$
$\overrightarrow{HD} = \langle 7,-3,0\rangle$

$\langle 4,0,-1\rangle \times \langle 7,-3,0\rangle$

$$\begin{vmatrix} i & j & k \\ 4 & 0 & -1 \\ 7 & -3 & 0 \end{vmatrix} = i\begin{vmatrix} 0 & -1 \\ -3 & 0 \end{vmatrix} - j\begin{vmatrix} 4 & -1 \\ 7 & 0 \end{vmatrix} + k\begin{vmatrix} 4 & 0 \\ 7 & -3 \end{vmatrix}$$

$-3i - 7j - 12k = \langle -3,-7,-12\rangle$ normal

porter: $\langle 2,2,1\rangle$

$\overrightarrow{HP} = \langle x-8, y-3, z-1\rangle$

$\langle -3,-7,-12\rangle \cdot \langle x-8, y-3, z-1\rangle = 0$

$-3(x-8) + -7(y-3) - 12(z-1) = 0$

$-3x+24 - 7y + 21 - 12z + 12$

$-3x - 7y - 12z = -57 \leftarrow$ our plane

$-3(2) - 7(2) - 12(1) =$

$-6 - 14 - 12$

$-32 \neq -57$ porter not on plane $\rightarrow$

$$\frac{A(x_1) + B(y_1) + C(z_1) + D}{\sqrt{A^2+B^2+Y^2}}$$

$$= \frac{-3(2) - 7(2) - 12(1) + 57}{\sqrt{(-3)^2+(-7)^2+(12)^2}}$$

$$= \frac{-32+57}{\sqrt{202}}$$

$$= \cancel{\frac{-89}{\sqrt{202}}} \quad \frac{25}{\sqrt{202}}$$

$$= \cancel{|-6.261|} \quad |1.76|$$

$$= \cancel{6.26\,m} \quad 1.76\,m$$

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