Calc I : Optimization
w/constraint

$$Area(L,W) = L \times W$$

$$Perim = 50 = 2L + 2W$$

$$W = \frac{50-2L}{2}$$

$$A(L) = L\left(\frac{50-2L}{2}\right)$$

$A = L \cdot W$ $\quad$ $2L + 2W = 50$

$\nabla A = \langle W, L \rangle$ $\quad$ $\nabla g = \langle 2, 2 \rangle$

$$\langle W, L \rangle = \lambda \langle 2, 2 \rangle$$

$W = 2\lambda$ $\quad$ $L = 2\lambda$

$2L + 2L = 50$ $\quad$ $L = 12.5$

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2. award: 10.00 points

Problems? Adjust credit for all students.

1 out of 3 attempts

Find the maximum and minimum of the function $f(x, y)$ subject to the constraint $g(x, y) \le c$.

$f(x, y) = 6x^2y$ subject to $x^2 + y^2 \le 15$

Maximum: ; Minimum:

Assistance: NetCalculator, Check My Work, View Hint, View Question, Show Me, Guided Solution, Practice This Question, Print, Question Help, Report a Problem

references

$$\nabla f = \langle 12xy, 6x^2 \rangle$$

$$\nabla g = \langle 2x, 2y \rangle$$

$$12xy = \lambda \cdot 2x$$

$$6x^2 = \lambda 2y$$

$$12xy = \frac{6x^2}{2y} \cdot 2x$$

$$12y^2 = 6x^2$$

$$y^2 = \frac{1}{2}x^2$$

$$y = \pm\sqrt{\frac{x^2}{2}}$$

$$x^2 + \frac{x^2}{2} \le 15$$

$$x^2 = 10$$

$$x = \pm\sqrt{10}$$

$$f(\sqrt{10}, \sqrt{5}) = 60\sqrt{5}$$

$$f(-\sqrt{10}, \sqrt{5}) = 60\sqrt{5}$$

$$f(\sqrt{10}, -\sqrt{5}) = -60\sqrt{5}$$

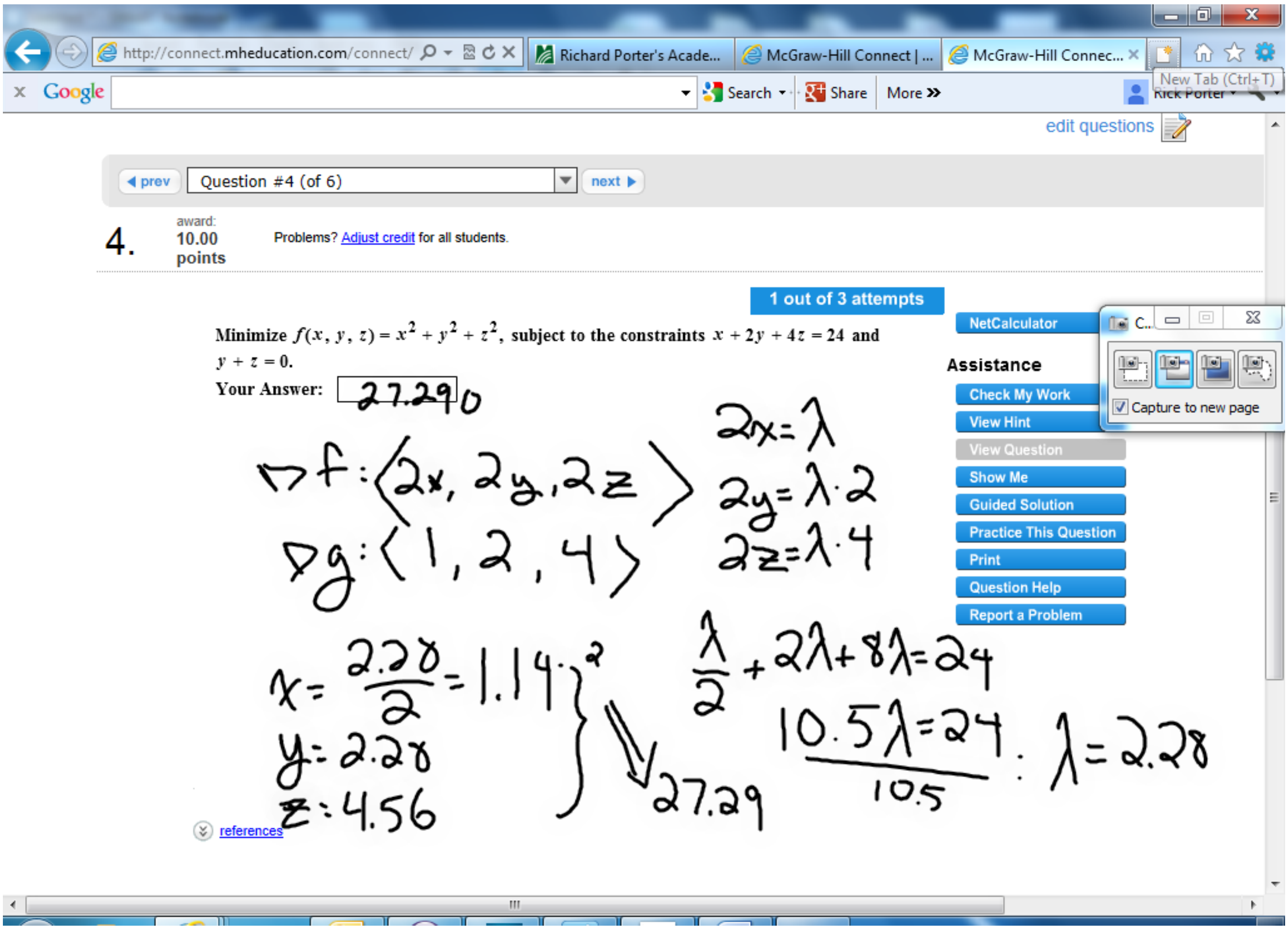
$$f(-\sqrt{10}, -\sqrt{5}) = -60\sqrt{5}$$

$$\nabla f = 0$$

$$\langle 12xy, 6x^2 \rangle = 0$$

$$x = 0$$

$$f(0, y) \quad f(0,y) = 0$$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next
4.
award:
10.00
points
Problems? Adjust credit for all students.
1 out of 3 attempts
Minimize f(x, y, z) = x^2 + y^2 + z^2, subject to the constraints x + 2y + 4z = 24 and y + z = 0.
Your Answer: 27.290
NetCalculator
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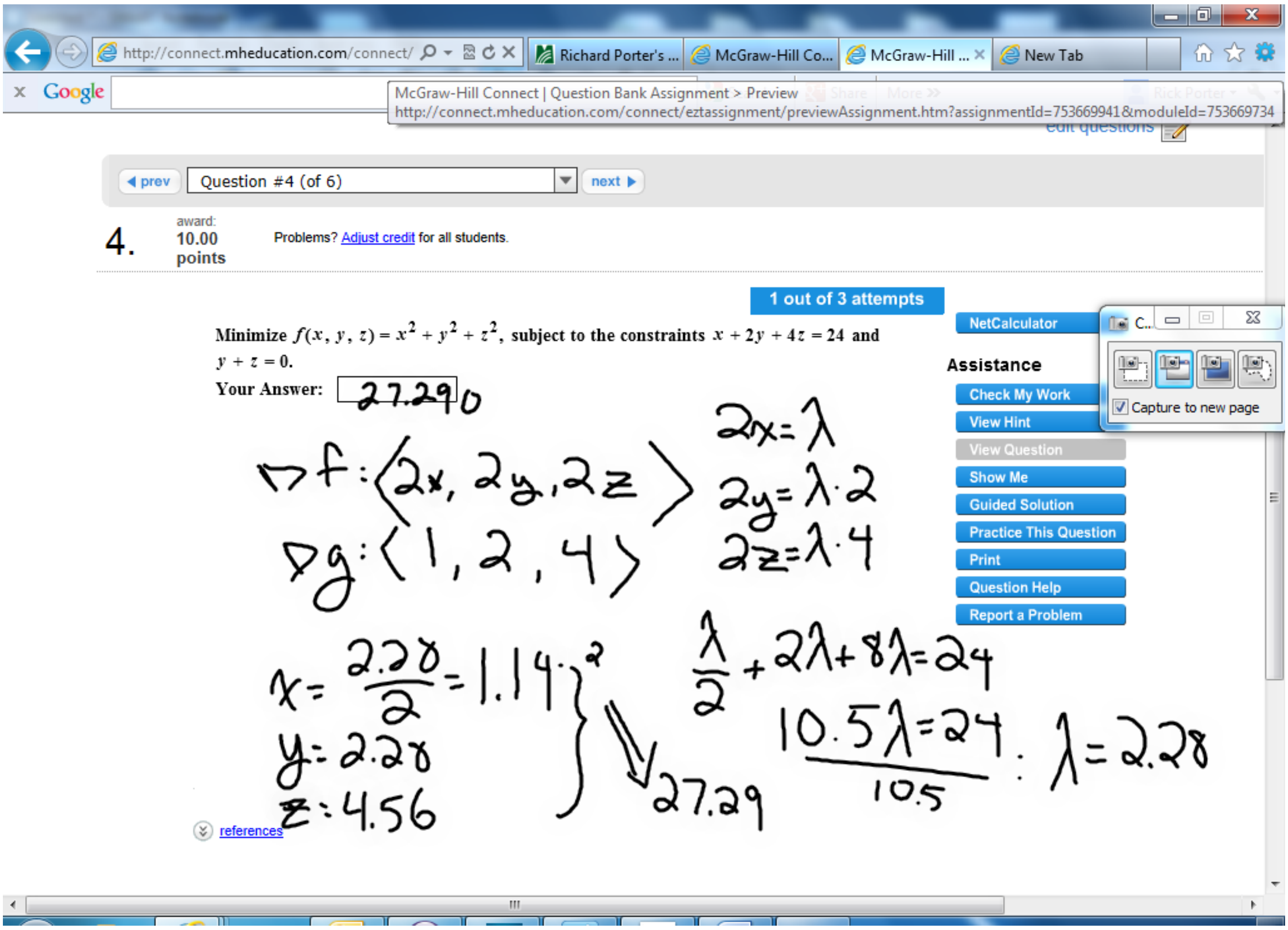
references
∇f: ⟨2x, 2y, 2z⟩
∇g: ⟨1, 2, 4⟩
2x = λ
2y = λ·2
2z = λ·4
λ/2 + 2λ + 8λ = 24
10.5λ = 24 : 10.5
λ = 2.28
x = 2.28/2 = 1.14
y = 2.28
z = 4.56
27.29



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Problems? Adjust credit for all students.
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Minimize $f(x, y, z) = x^2 + y^2 + z^2$, subject to the constraints $x + 2y + 4z = 24$ and $y + z = 0$.
Your Answer: 27.29
NetCalculator
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references
$\nabla f: \langle 2x, 2y, 2z \rangle$
$\nabla g: \langle 1, 2, 4 \rangle$
$2x = \lambda$
$2y = \lambda \cdot 2$
$2z = \lambda \cdot 4$
$\frac{\lambda}{2} + 2\lambda + 8\lambda = 24$
$10.5\lambda = 24$; $\lambda = 2.28$
$x = \frac{2.28}{2} = 1.14$
$y = 2.28$
$z = 4.56$
27.29

Show Me

Minimize $f(x, y, z) = x^2 + y^2 + z^2$, subject to the constraints $x + 2y + 4z = 24$ and $y + z = 0$.

h:

$$z + -2z + 4z = 24$$

$$3z = 24$$

$$z = 8$$

$$f(x, y, z) = x^2 + y^2 + z^2$$
$$g(x, y, z) = x + 2y + 4z - 24 = 0$$
$$h(x, y, z) = y + z = 0$$
$$\nabla f = \langle 2x, 2y, 2z \rangle$$
$$\nabla g = \langle 1, 2, 4 \rangle$$
$$\nabla h = \langle 0, 1, 1 \rangle$$

$$z = 2x + y$$

$$-z$$

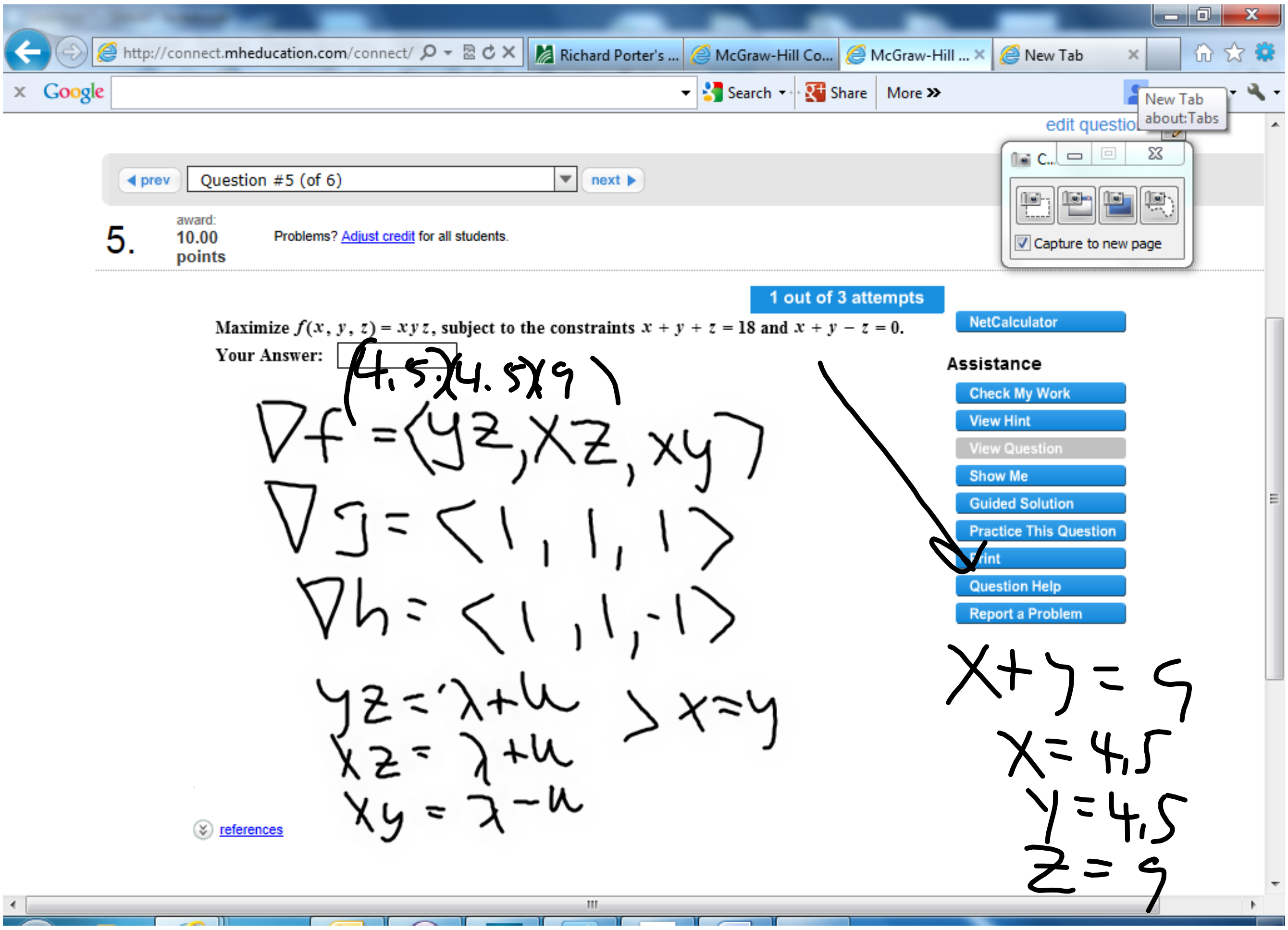
Setting $\nabla f = \lambda \nabla g + \mu \nabla h$ gives the equations:

$$2x = \lambda$$
$$2y = 2\lambda + \mu$$
$$2z = 4\lambda + \mu$$

$$2z = 2x$$

$$z = x$$

The first and second equations give $\lambda = 2x$ and $\mu = 2y - 4x$.
Then the third equation yields $z = 2x + y$.
Substituting this into $h(x, y, z) = 0$ gives $x = -y$, and using these relationships in $g(x, y, z) = 0$ then shows $y = 8$, $z = -8$ and $x = -8$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5.
award:
10.00
points
Problems? Adjust credit for all students.
1 out of 3 attempts
Maximize $f(x, y, z) = xyz$, subject to the constraints $x + y + z = 18$ and $x + y - z = 0$.
Your Answer: (4.5)(4.5)(9)
$\nabla f = \langle yz, xz, xy \rangle$
$\nabla g = \langle 1, 1, 1 \rangle$
$\nabla h = \langle 1, 1, -1 \rangle$
$yz = \lambda + u$
$xz = \lambda + u$
$xy = \lambda - u$
$x = y$
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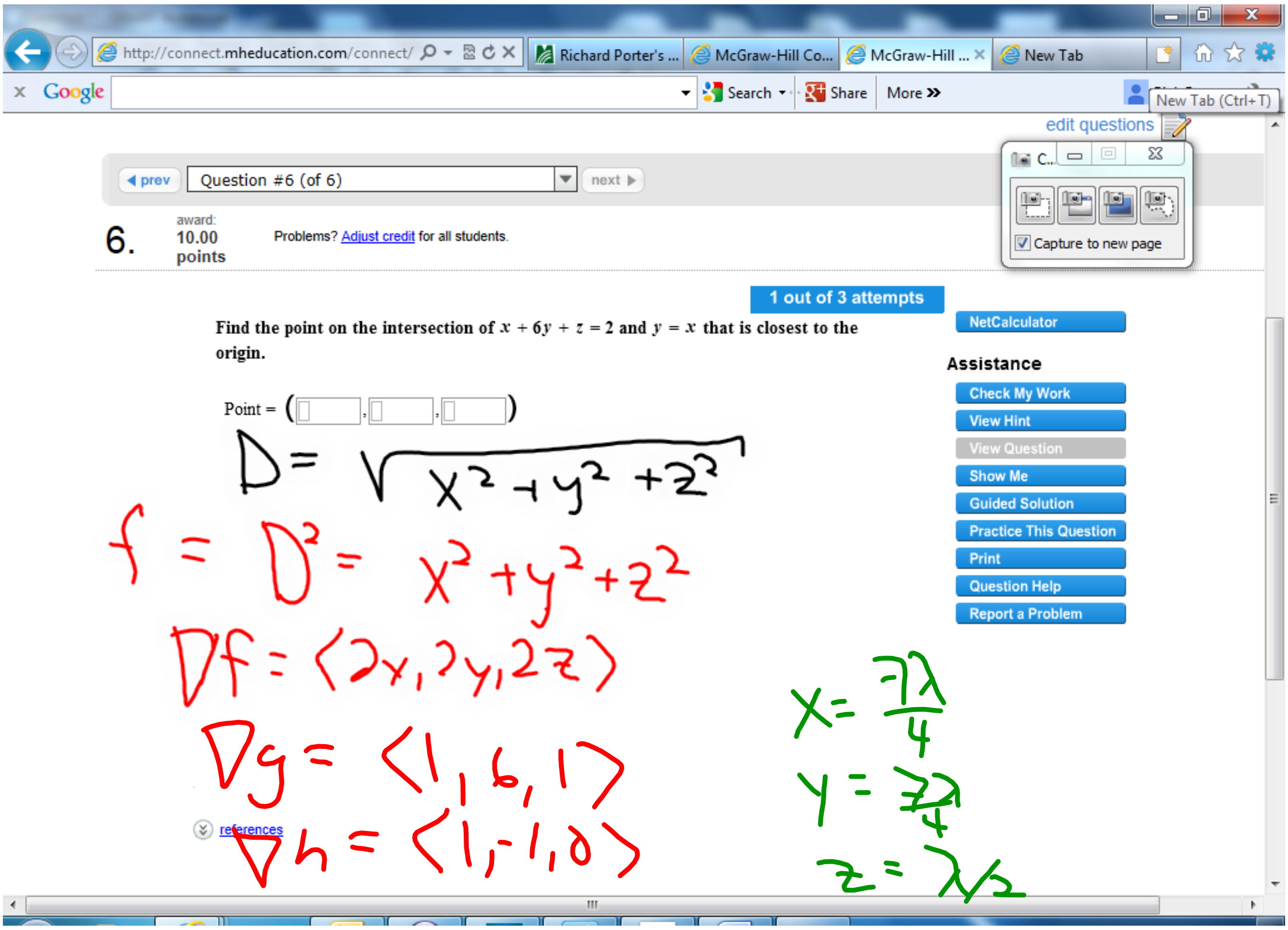
$x + y = 9$
$x = 4.5$
$y = 4.5$
$z = 9$
references



$D = \sqrt{x^2 + y^2 + z^2}$

$f = D^2 = x^2 + y^2 + z^2$

$\nabla f = \langle 2x, 2y, 2z \rangle$

$\nabla g = \langle 1, 6, 1 \rangle$

$\nabla h = \langle 1, -1, 0 \rangle$

$x = \frac{7\lambda}{4}$

$y = \frac{7\lambda}{4}$

$z = \lambda/2$

$2x = \lambda + \mu$

$2y = 6\lambda - \mu$

$2z = \lambda$

$4x = 7\lambda$

$\frac{4x}{7} = \lambda$

center building at origin

at least 10 stories tall

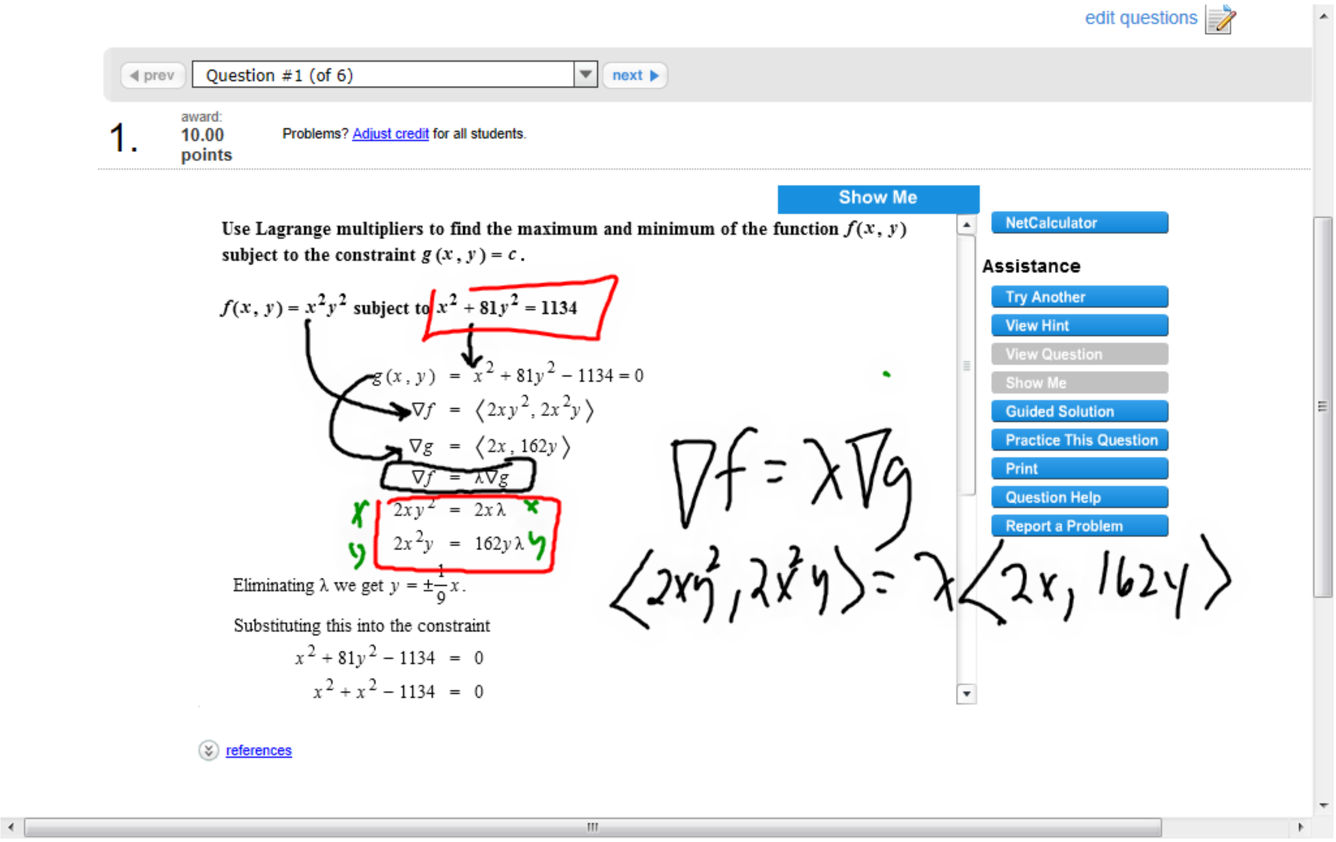
dome on top

somewhere underneath the dome, but in Quadrant 1...hole in dome. Which direction does a marble pushed out of the hole go?

Find the gradient at that point.

$x^2+y^2+(z-100)^2=100$

$z(x,y)=(100-x^2-y^2)^{.5}+100$



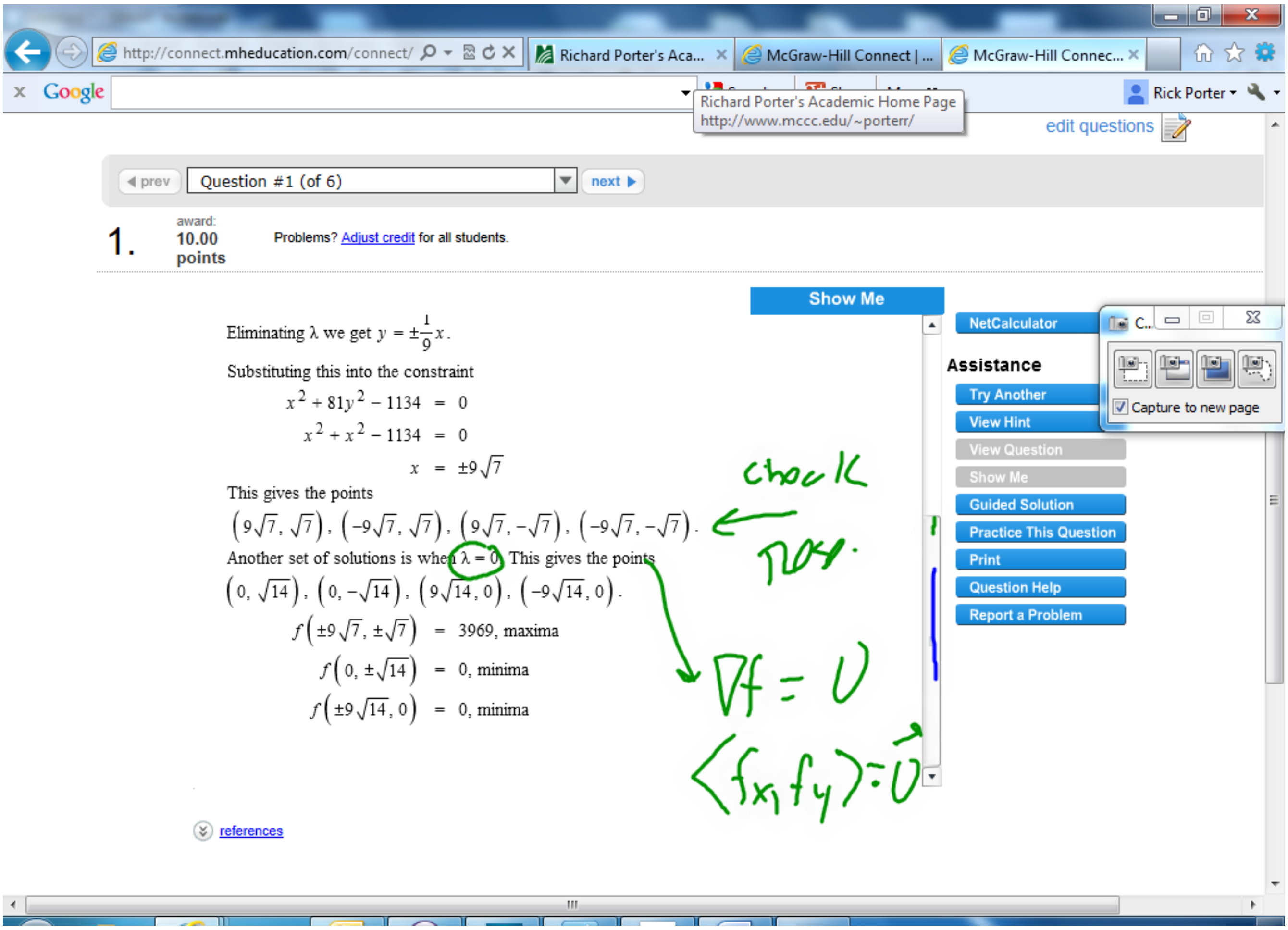
$-81 \cdot 2xy^2 = 2x\lambda \quad -81 \rightarrow \quad -162y^2 = 162\lambda$

$2x^2 y = 162 y \lambda \qquad 2x^2 = 162\lambda$

$x^2 + 81\left(\frac{x}{9}\right)^2 - 1134 = 0 \qquad -162y^2 + 2x^2 = 0$

$2x^2 = 1134 \qquad x^2 = 81y^2$

$x = \pm 9y \qquad y = \pm\frac{1}{9}x$



Question #1 (of 6)

1. award: 10.00 points

Problems? Adjust credit for all students.

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Eliminating λ we get $y = \pm\frac{1}{9}x$.

Substituting this into the constraint

$$x^2 + 81y^2 - 1134 = 0$$
$$x^2 + x^2 - 1134 = 0$$
$$x = \pm 9\sqrt{7}$$

This gives the points

$\left(9\sqrt{7}, \sqrt{7}\right)$, $\left(-9\sqrt{7}, \sqrt{7}\right)$, $\left(9\sqrt{7}, -\sqrt{7}\right)$, $\left(-9\sqrt{7}, -\sqrt{7}\right)$.

Another set of solutions is when $\lambda = 0$. This gives the points

$\left(0, \sqrt{14}\right)$, $\left(0, -\sqrt{14}\right)$, $\left(9\sqrt{14}, 0\right)$, $\left(-9\sqrt{14}, 0\right)$.

$$f\left(\pm 9\sqrt{7}, \pm\sqrt{7}\right) = 3969, \text{ maxima}$$
$$f\left(0, \pm\sqrt{14}\right) = 0, \text{ minima}$$
$$f\left(\pm 9\sqrt{14}, 0\right) = 0, \text{ minima}$$

check

now.

$\nabla f = \vec{0}$

$\langle f_x, f_y \rangle = \vec{0}$

references