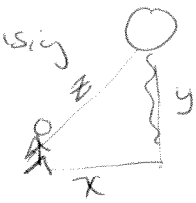
Related Rates (Balloon problems)



Filling

Sell

Heu

① $x^2+y^2=z^2$

Equation relating variable

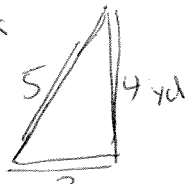
② Derivative with respect to "t"

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$$

③ Identify unknowns

Values	Rates
$x=$	$dx/dt=$
$y=$	$dy/dt=$
$z=$	$dz/dt=$

Ex



If balloon rises at 8 yds/sec

How Fast is the distance between You changing?

$x=3$ $dx/dt=0$

$y=4$ $dy/dt=8$

$z=5$ $dz/dt=?$

$2(3)(0)+2(4)(8)=2(5)\frac{dz}{dt}$ 6.4 yds/sec

$\frac{dx}{dt}>0$ walk away

$\frac{dx}{dt}<0$ walk twds

$\frac{dx}{dt}=0$ stand.

$\frac{dy}{dt}>0$ balloon Rising

$\frac{dz}{dt}=$ ~~rate~~ ~~distan~~ between you/balloon

① $V=\frac{4}{3}\pi r^3$

② $\frac{dV}{dt}=\frac{4}{3}\pi 3r^2\frac{dr}{dt}$

③

r = radius value

$\frac{dr}{dt}$ = growing radius

$\frac{dV}{dt}$ = growing volume

Ex Fill balloon at 80cc/sec = rate

How fast is the radius change when r=8, r=10 values

$\frac{dV}{dt}=80\frac{cc}{sec}=4\pi r^2\frac{dr}{dt}$

$\frac{80}{4\pi r^2}=\frac{4\pi r^2}{4\pi r^2}\frac{dr}{dt}$

r=8 smaller

$\frac{dr}{dt}=\frac{80}{4\pi r^2}=\frac{80}{4\pi\cdot 64}=.099\ldots\frac{cm}{sec}$

r=10 ~~bigger~~

$\frac{dr}{dt}=\frac{80}{4\pi(10)^2}=.064$ cm/sec

Balloon is growing slower as radius gets bigger

$R=P\times S$

$\frac{dR}{dt}=P\frac{dS}{dt}+S\frac{dP}{dt}$

P = price, S = Sales

$\frac{dS}{dt}$ = sales are changing

$\frac{dP}{dt}$ = price is changing

Ex How much is revenue changing if a $5 balloon sell at 12 units raise price by $.50 a day loose 1 customer a day.

$P=\$5$ $dP/dt=.50$

$S=12$ $dS/dt=-1$

$R=\$60$

$\frac{dR}{dt}=(5)(-1)+12(.50)$

$=-5+6=\$1$/day

NEXT DAY

$P=\$6$ $dP/dt=.50$

$S=11$ $dS/dt=-1$

$\frac{dR}{dt}=(6)(-1)+11(.50)$

$=-6+5.5=-.50$/day

Project.

$y=f(x)$

$\frac{dy}{dt}=f'(x)\cdot\frac{dx}{dt}$

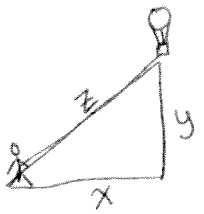
Related Rates

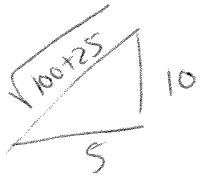
Balloon Rising

Triangle Pythagorean Theorem $x^2 + y^2 = z^2$

Take derivative d/dt $\frac{d}{dt}(x^2+y^2) = \frac{d}{dt} z^2$

$$2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right) = 2z\left(\frac{dz}{dt}\right)$$





Values x, y, z

Rising at 6 ft/sec $= \frac{dy}{dt}$

Walking towards ballon .5/sec $= dx/dt$

$x = 5$ $dx/dt = -.5$

$y = 10$ $dy/dt = 6$

$z = \sqrt{125}$ $dz/dt = ?$

How fast is the distance between you and balloon changing? $dz/dt = ?$

when I'm 5' from the start of balloon and

The balloon is 10' high.

$x = 5$

$y = 10$

$$\frac{2(5)(-.5) + 2(10)(6)}{2(\sqrt{125})} = \frac{dz}{dt}$$

$5.14 = dz/dt$

Filling Balloon

$V = \frac{4}{3}\pi r^3$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Values: r

Rates: $\frac{dr}{dt}$, $\frac{dV}{dt}$

Selling Balloon

$R = P \times S$

$$\frac{dR}{dt} = P \cdot \frac{dS}{dt} + S \cdot \frac{dP}{dt}$$

Ex You can sell \$5 you can sell 12 balloon

You decide lower yor price by \$.10 per week

You sell more balloons at .25 per week.

How is revenue changing

VALUE

$P = \$5$ $-\$.10/\text{wk} = dP/dt$

$S = 12$ $+.25/\text{wk} = dS/dt$

$R = 60$

$$\frac{dR}{dt} = 5 \cdot (.25) + 12(-.1)$$

$$= 1.25 - 1.2 = .05$$