Homework #7

Empire Loquan Drummer Fabian Best

$$4\cos(x)\sin(y)=1$$

$fg'+f'g$ $\quad 0=(4\cos(x))(\cos(y))\frac{dy}{dx}+(\sin(y))(-4\sin(y))$

$$\frac{dy}{dx}=\frac{-(\sin(y))(-4\sin(y))}{(\cos(y))(-4\cos(y))}$$

FR3CH

3.5 #10/213: $y^5 + x^2y^3 = 1 + ye^{x^2}$

Differentiate both sides of the equation:

$$\frac{d}{dx}(y^5 + x^2y^3) = \frac{d}{dx}(1 + ye^{x^2})$$

$$\frac{d}{dx}(y^5) + \frac{d}{dx}(x^2y^3) = \frac{d}{dx}(1) + \frac{d}{dx}(ye^{x^2})$$

We have:

+ $$\frac{d}{dx}(y^5) = 5y^4\frac{dy}{dx}$$

+ $$\frac{d}{dx}(x^2y^3) = \frac{d}{dx}(x^2)\cdot y^3 + x^2\frac{d}{dx}(y)^3$$
$$= 2xy^3 + x^2\,3y^2\frac{dy}{dx}$$

+ $$\frac{d}{dx}(1) = 0$$

+ $$\frac{d}{dx}(ye^{x^2}) = \frac{d}{dx}(y)\cdot e^{x^2} + \frac{d}{dx}(e^{x^2})\cdot y$$
$$= \frac{dy}{dx}\cdot e^{x^2} + 2xe^{x^2}\cdot y$$

Thus:

$$5y^4\frac{dy}{dx} + 2xy^3 + x^2\,3y^2\frac{dy}{dx} = 0 + \frac{dy}{dx}e^{x^2} + 2xye^{x^2}$$

$$\Leftrightarrow 5y^4\frac{dy}{dx} + x^2\,3y^2\frac{dy}{dx} - e^{x^2}\frac{dy}{dx} = 2xye^{x^2} - 2xy^3$$

$$\Leftrightarrow (5y^4 + x^2\,3y^2 - e^{x^2})\frac{dy}{dx} = 2xye^{x^2} - 2xy^3$$

$$\Leftrightarrow \frac{dy}{dx} = \frac{2xye^{x^2} - 2xy^3}{5y^4 + 3x^2y^2 - e^{x^2}}$$

Science Buddies

Section 3.4
#24

$$y \sec x = x \tan y$$

$$fg' + gf' = fg' + gf'$$

$$y \frac{d}{dx}(\sec x) + \sec x \frac{d}{dx}(y) = x \frac{d}{dx}(\tan y) + \tan y \frac{d}{dx}(x)$$

$$y(\sec x \tan x) + \cancel{\sec x\, y'} = x \sec^2 y\, y' + \tan y\, x'$$
$$\quad - \cancel{\sec x\, y'} \qquad\qquad\qquad - \sec x y'$$

*Now we put y' on one side. $\boxed{x' = 1}$

$$y(\sec x \tan x) = x \sec^2 y\, y' - \sec x\, y' + \cancel{\tan y}$$
$$-\tan y \qquad\qquad\qquad\qquad - \cancel{\tan y}$$

$$y(\sec x \tan x) - \tan y = x \sec^2 y\, y' - \sec x\, y'$$

$$\frac{y(\sec x \tan x) - \tan y}{\alpha(\sec^2 y - \sec x)} = y' \frac{\cancel{(x \sec^2 y - \sec x)}}{\cancel{(x \sec^2 y - \sec x)}}$$

$$\Rightarrow \quad y' = \frac{y(\sec x \tan x) - \tan y}{x(\sec^2 y - \sec x)}$$

team: C • A • M •

SECTION
3.5

#36

$$x^4 + y^4 = a^4$$

$$4x^3 + 4y^3y' = 0$$

$$\frac{dy}{dx} = \frac{-4x^3}{4y^3}$$

$$\frac{dy}{dx} = \frac{-x^3}{y^3}$$

$$\frac{d^2y}{dx^2} = \frac{-y^3(3x^2) + x^3(3y^2)\frac{dy}{dx}}{y^6}$$

$$= \frac{-3x^2y^3 + 3x^3y^2\left(\frac{-x^3}{y^3}\right)}{y^6}$$

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Pythagorus

50 $y = \tan^{-1}\left(x - \sqrt{1+x^2}\right)$

$y' = \dfrac{1}{1+\left(x-\sqrt{1+x^2}\right)^2} \cdot 1 - \dfrac{1}{2}\left(1+x^2\right)^{-\frac{1}{2}} \cdot 2x$

$= \dfrac{1}{1+\left(x - \sqrt{1+x^2}\right)^2} \cdot \left(1 - \dfrac{x}{\sqrt{1+x^2}}\right)$

$= \dfrac{1}{1+x^2-2x\sqrt{x^2+1}+x^2+1} \left(\dfrac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}}\right)$

$= \dfrac{\sqrt{x^2+1}-x}{2\left(1+x^2-x\sqrt{x^2+1}\right)\sqrt{x^2+1}}$

$= \dfrac{\sqrt{x^2+1}-x}{2\left[\sqrt{x^2+1}\left(1+x^2\right)-x\left(x^2+1\right)\right]}$

$= \dfrac{\sqrt{x^2+1}-x}{2\left[\left(1+x^2\right)\left(\sqrt{x^2+1}-x\right)\right]}$

Mike Gankhuyag
Guan Zhang

MAT151... { Team α } 2/28/10
3.6) #12

Jonathan Chen

12.) $y = \ln(x + \sqrt{x^2 - 1})$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\ln(x + \sqrt{x^2-1})\right]$$

$$= \frac{1}{(x+\sqrt{x^2-1})} \frac{d}{dx}\left(x + \sqrt{x^2-1}\right)$$

$$= \frac{1}{(x+\sqrt{x^2-1})} \frac{d}{dx}(x) + \left[\frac{d}{dx}(\sqrt{x^2-1}) \Rightarrow \frac{1}{2}(x^2-1)^{(-1/2)}\right]$$

$$= \frac{1}{(x+\sqrt{x^2-1})} \left[1 + \frac{1}{2}(x^2-1)^{(-1/2)} \frac{d}{dx}(x^2-1)\right]$$

$$= \frac{1}{(x+\sqrt{x^2-1})} \left[1 + \frac{1}{2}(x^2-1)^{(-1/2)}(2x)\right]$$

$$= 1 + (x/\sqrt{x^2-1})$$

$$= \frac{1 + (x/\sqrt{x^2-1})}{(x+\sqrt{x^2-1})}$$

B.A is B.S

Will Arbito Ryan Zhao

31. $f(x) = \dfrac{\ln x}{x^2}$

$f'(x)\ \dfrac{x^2 \cdot \frac{1}{x} - 2x \cdot \ln x}{(x^2)^2}$

$f'(x) = \dfrac{\frac{x^2}{x} - 2x \ln x}{x^4}$

$f'(1) = \dfrac{1 - 0}{1}$

$f'(1) = 1$

Team Diesel 3.6 #45

Stanley Tuchez, Tyler Ferst, Connor Payne 3/1/10

USE Logarithmic Differentiation

$y = (\cos x)^x$

$\ln y = \ln((\cos x)^x)$ - Property of logs

$\ln y = x \ln \cos x$

Then, use product rule & chain Rule (take derivative)

Multiply both sides by y to get the derivative alone.

$\left(\frac{1}{y} y' = x\left(\frac{1}{\cos x}\right)(-\sin x) + (\ln \cos x)1\right)$ ⇒ Now simplify

$y' = y\left(\ln \cos x - \frac{x \sin x}{\cos x}\right)$

$y = (\cos x)^x$ $\quad \frac{-\sin x}{\cos x} = -\tan x$

So ⇒

$y' = (\cos x)^x(\ln \cos x - x \tan x)$

Team Kickass

3.6 #46

Find the derivative of the function

$y = \sqrt{\tan^{-1} x}$

$y' = (\tan^{-1} x)^{\frac{1}{2}} \cdot \frac{d}{dx} \tan^{-1} x$ (take the derivative of the inner function)

$y' = \frac{1}{2}(\tan^{-1} x)^{-\frac{1}{2}} \cdot \left(\frac{1}{1+x^2}\right)$

(bring out the $\frac{1}{2}$)

$(1 - \frac{1}{2})$
$= -\frac{1}{2}$