

Homework #15

Laquan Drummer
Sai Janardhan
Fabian Best
EMPIRE

4.5^F

Sketch

$$\frac{x}{x^3-1}$$

Using guidelines

A. domain $(-\infty, 1) \cup (1, \infty)$

B. $f(0) = \frac{0}{0-1} = 0$ y-intercept = 0

$\frac{x}{x^3-1} = 0$, math solver x-intercept = 0

C. $\frac{-x}{-x^3-1} = \text{odd function}$

D. $\lim_{x \rightarrow \infty} \frac{x}{x^3-1} = 0$ horizontal asymptote $va = 1$

E. $f'(x) = \frac{x}{x^3-1} = \frac{(x^3-1)(1) - (x)(3x^2)}{(x^3-1)^2} = \frac{x^3-1-3x^3}{(x^3-1)^2}$

$$\begin{array}{c|ccc} -2 & 0 & 1 \\ \hline | & - & \emptyset & - & | \end{array}$$

$$\frac{-1-1-3}{(-1-1)^2} = \frac{-5}{(-2)^2} = \frac{-5}{4} = -1.25$$

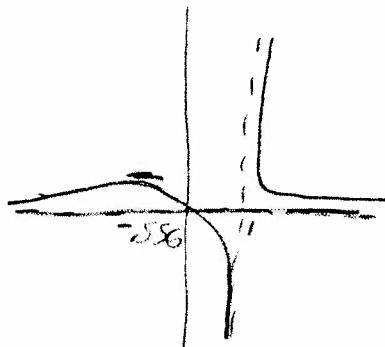
F. $f'(x) = 0$

$$\frac{x^3-1-3x^3}{(x^3-1)^2} = 0$$

Math solver

$$x = 1.879$$

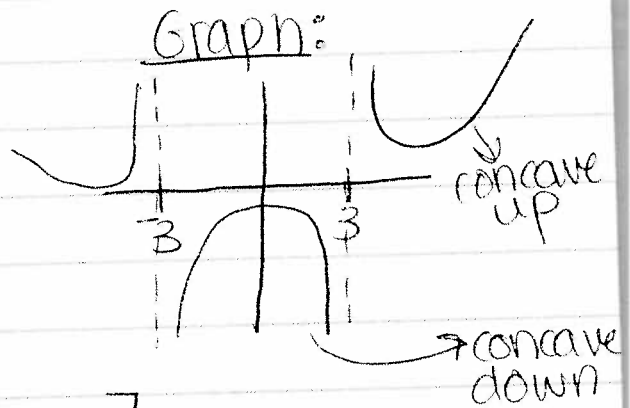
$$x = -.556$$



Team Kickass

4.6 #8

$$f(x) = \frac{e^x}{x^2 - 9}$$



$$f'(x) = \left[\frac{(x^2 - 9)e^x - (2x)e^x}{(x^2 - 9)^2} \right] = 0$$

$$= x^2 e^x - 9e^x - 2x e^x = 0$$

$$= e^x (x^2 - 2x - 9) = 0$$

use Quadratic Equation

$$= \frac{2 \pm \sqrt{4 - 4(-9)}}{2}$$

$$= 1 \pm \sqrt{10}$$

$$f''(x) =$$

TEAM: C.A.M.

~~4.5~~

4.7

#4

Find a positive number such that the sum of the number and its reciprocal is as small as possible.

$$f(x) = x + \frac{1}{x}$$

$$f'(x) = 1 + \left(-\frac{1}{x^2}\right)$$

$$0 = 1 - \left(\frac{1}{x}\right)^2$$

$$-1 = -\left(\frac{1}{x}\right)^2$$

$$\sqrt{x^2} = \sqrt{1}$$

$$x = \pm \sqrt{1}$$

$$x = 1$$

$$f(1) = (1) + \frac{1}{(1)}$$

$$= 2$$

CASANDRA WERO
Augustin Ciocotisan
MAYA HUSABAYAR

4.7
#6

$$\text{Area} = \text{Length} * \text{Width}$$

$$\text{area} = 1000$$

$$\text{Perimeter} = 2(L) + 2(w)$$

$$\frac{1000}{L} = \text{width}$$

$$P = 2(L) + 2\left(\frac{1000}{L}\right)$$

$$P = 2(L) + \frac{2000}{L}$$

$$P1 = 2 - 2000 L^{-1}$$

$$2 = 2000 / L^2$$

$$L^2 = 1000$$

$$L = \sqrt{1000}$$

THE NEW
GROUP

Kristian, Melua, Talisha
4.7 #8

Save The Polar Bears

$$P = \frac{100I}{I^2 + I + 4}$$

Use Quotient Rule!

$$P' = \frac{I^2 + I + 4 \frac{d}{dI}(100I) - 100I \frac{d}{dI}(I^2 + I + 4)}{(I^2 + I + 4)^2}$$

$$P' = \frac{I^2 + I + 4(100) - 100I(2I + 1)}{(I^2 + I + 4)^2}$$

$$P' = \frac{100I^2 + 100I + 400 - 200I^2 - 100I}{(I^2 + I + 4)^2}$$

$$P' = \frac{-100I^2 + 400}{(I^2 + I + 4)^2}$$

Set numerator equal to zero to find critical value where the function is at its max!

$$-100I^2 + 400 = 0$$

$$\frac{-100I^2}{-100} = \frac{-400}{-100}$$

$$I^2 = 4$$

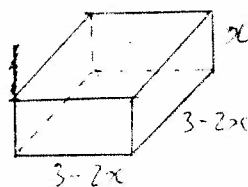
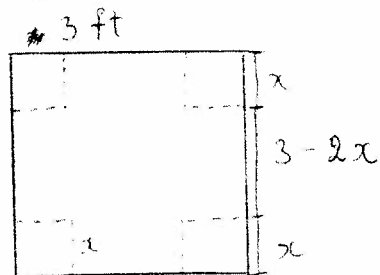
$$I = 2$$

∴ P is a maximum at light intensity

2!

FR3CH

4.7 # 10



(open top)

Find the largest volume of a box^A made from a square piece of cardboard 3 ft wide, by cutting out a square from each of 4 corners and bending up sides.

Solution

$$\begin{aligned} \text{Volume: } V &= (3 - 2x)(3 - 2x)x \\ &= 4x^3 - 12x^2 + 9x \end{aligned}$$

Derivative of V :

$$V' = 12x^2 - 24x + 9$$

$$V' = 0 \quad (\Leftrightarrow) \quad 12x^2 - 24x + 9 = 0$$

$$(\Leftrightarrow) \quad \begin{cases} x = 1/2 \\ x = 3/2 \end{cases}$$

$$+ \quad x = \frac{1}{2} : V = (3 - 2 \cdot \frac{1}{2})(3 - 2 \cdot \frac{1}{2}) \cdot \frac{1}{2} = 2 \text{ ft}^3 \quad (\text{max})$$

$$+ \quad x = \frac{3}{2} : V = (3 - 2 \cdot \frac{3}{2})(3 - 2 \cdot \frac{3}{2}) \cdot \frac{3}{2} = 0 \text{ ft}^3 \quad (\text{min})$$

The largest volume of a box is 2 ft^3

CIVARC

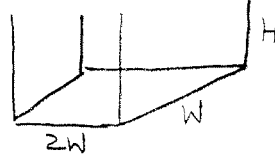
J. K. HOFF

R. D'ODIA

S. MOWLE

SECTION 4.70

14. $V = 10 \text{ m}^3$
 $V = L \times W \times H$
 $V = (2W)(W)(H)$
 $10 \text{ m}^3 = 2W^2(H)$
 $\frac{10 \text{ m}^3}{2W^2} = H$



COST SIDE = $\$6/\text{m}^2 = 2(6 \times WH) = 12WH$
 $2[\$6(2W)(H)] = 24WH$

COST BASE = $\$10/\text{m}^2 = 2W(W)(\$10/\text{m}^2) = 20W^2$

$C = 36WH + 20W^2$

$C = 36W\left(\frac{10}{2W^2}\right) + 20W^2$

$C = \frac{180}{W} + 20W^2$

$C' = -180W^{-2} + 40W = 0$

$\frac{180}{W} = 40W$

$\frac{18}{4} = W^2$

$4.50 = W^2$

$\sqrt{4.50} = 2.121 \dots$

$W = 2.12$

$H = \frac{10}{2(2.12)^2} = 1.112 \dots$

$C = 36(2.12)(1.11) + 20(2.12)^2$

$C = 84.72 + 89.89$

$C = \$174.60$

BAI > BS

4.7.18

Ryan ZHAO
Tiejun Wen

$$6x + y = 9$$

$$1 = (-3, 1)$$

$$y = 9 - 6x$$

$$D^2 = (x+3)^2 + (y-1)^2 =$$

$$= (x+3)^2 + (9-6x-1)^2$$

$$= x^2 + 6x + 9 + 64 + 36x^2 - 96x$$

$$= 37x^2 - 90x + 73$$

$$(D^2)' = 74x - 90 = 0$$

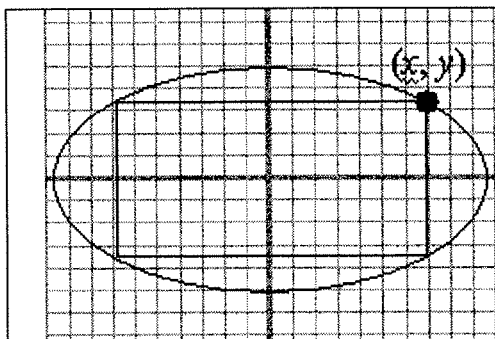
$$x = \frac{90}{74} = \frac{45}{37}$$

$$y = 9 - 6\left(\frac{45}{37}\right) = \frac{63}{37}$$

$$\text{Point } \left(\frac{45}{37}, \frac{63}{37}\right)$$

Grundle Pumpkins

Pages 329 4.7.22 Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.



Solve i(Simple):

$$\because a^2 + b^2 \geq 2ab(x, y, a, b > 0)$$

$$\therefore 1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} \geq 2\sqrt{\frac{x^2}{a^2} \cdot \frac{y^2}{b^2}} = \frac{2xy}{ab}$$

$$\therefore xy \leq \frac{ab}{2}$$

$$\therefore A = 4xy \leq 4 \cdot \frac{ab}{2} = 2ab$$

Solve ii(Calculus):

the length of this rectangle will be $2x$, and the width of this rectangle will be $2y$

$$\because x, y, a, b > 0$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow y^2 = b^2 \left(1 - \frac{x^2}{a^2}\right) \Rightarrow y = b\sqrt{1 - \frac{x^2}{a^2}}$$

$$A = 2x \cdot 2y = 2x \cdot 2b\sqrt{1 - \frac{x^2}{a^2}} = \frac{4bx}{a}\sqrt{a^2 - x^2}$$

$$A' = \frac{4b}{a} \left(x \cdot \frac{1}{2} (a^2 - x^2)^{\left(-\frac{1}{2}\right)} (-2x) + \sqrt{a^2 - x^2} \right)$$

$\because A' = 0$ (set the derivative equal to zero)

$$\therefore 0 = \frac{4b}{a} \left(\frac{x}{2} (-2x) + (a^2 - x^2) \right) = 4ab - \frac{8b}{a} x^2 \Rightarrow x = \frac{a}{\sqrt{2}}$$

$$\therefore A = \frac{4b}{a} \frac{a}{\sqrt{2}} \sqrt{a^2 - \left(\frac{a}{\sqrt{2}}\right)^2} = \frac{4b}{\sqrt{2}} \sqrt{a^2 - \left(\frac{a}{\sqrt{2}}\right)^2} = \frac{4b}{\sqrt{2}} \sqrt{\frac{a^2}{2}} = 2ab$$

Sam Tobia

Pablo Espichan

Tiejun Wen

4.7

We Love Math

$$40. \quad F = \frac{\mu W}{\mu \sin \theta + \cos \theta}$$

$$\frac{d}{dx} (\mu W) (\mu \sin \theta + \cos \theta)^{-1}$$

$$\mu W (-1) (\mu \sin \theta + \cos \theta)^{-2} (\mu \cos \theta - \sin \theta) = 0$$

has to be 0

$$\frac{\mu \cos \theta}{\cos \theta} = \frac{\sin \theta}{\cos \theta}$$

$$u = \tan \theta$$

$$\theta = \tan^{-1} u$$

LWV

4.7 # 53

WILGENS
VIOLENE

$$C(x) = \frac{C(x)}{x}$$

$$C(x) = 16000 + 200x + 4x^{3/2}$$

$$x = 1000 \text{ units}$$

$$C(1000) = 16000 + 200(1000) + 4(1000)^{3/2}$$

$$C(1000) = \underline{\underline{\$ 342,491}}$$

$$C(x) = \frac{C(x)}{x} \Rightarrow C'(x) = \frac{x C'(x) - C(x)}{x^2} = 0$$

$$x C'(x) - C(x) = 0 \Rightarrow x C'(x) = C(x)$$

$$C'(x) = \frac{C(x)}{x}$$

$$C'(x) = \frac{342491}{1000} \Rightarrow \underline{\underline{A.C = 342.49/\text{unit}}}$$

$$C'(x) = 200 + 6x^{1/2} = 200 + 6(1000)^{1/2}$$

$$\underline{\underline{M.C = 389.74/\text{units}}}$$

(ii) Production level $C'(x) = 0$

that will

minimize the A.C.

$$200 + 6x^{1/2} = 0$$

$$6x^{1/2} = -200 \Rightarrow x^{1/2} = \left(-\frac{200}{6}\right)^2$$

$$x = (33.33)^2 = 1111.$$



~~HWV~~ HWV

4.7
#53

$$C(x) = \frac{16000 + 200(1111) + 4(1111)^{3/2}}{1111}$$

$$= \frac{27525.348}{1111}$$

$$= \$ 247.75 \text{ is minimum Average Cost.}$$

54.) a.) show that if the profit $P(x)$ is a max, then the marginal revenue equals the marginal cost.

□ total profit: $P(x) = R(x) - C(x)$

marginal rev: $R'(x)$; $\left[R(x) = x p(x) \Rightarrow R'(x) = \overset{\text{product rule}}{x p'(x) + p(x)} \right]$

marginal cost: $C'(x)$

ans: $\nabla P(x)$ is a max: $P'(x) = R'(x) - C'(x) = 0 \Rightarrow \boxed{R'(x) = C'(x)}$
 $x p'(x) + p(x) = C'(x)$

b.) if $[C(x) = 16000 + 500x - 1.6x^2 + 0.004x^3]$ is the cost function & $[p(x) = 1700 - 7x]$ is the demand function, find the production lvl. that will maximize profit.

□ $R(x) = x p(x)$, $p(x) = 1700 - 7x$: $R(x) = x [1700 - 7x]$

$R(x) = 1700x - 7x^2 \Rightarrow [R'(x) = 1700 - 14x = 0] \Rightarrow x = 121.4$

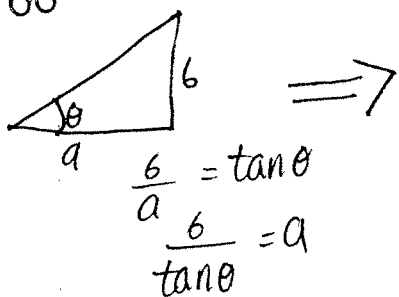
$C(121.4) = 16000 + 500(121.4) - 1.6(121.4)^2 + 0.004(121.4)^3$
 $= 16,000 + 60,700 - 23,580.74 + 7156.75$
 $= 76,700 - 23,580.74 + 7156.75$
 $= 53,119.26 + 7156.75$
 $= \boxed{60,276.01}$

★ 60,276.01 is the production that will max. profit.

Mike Gankhuyag
Jonathan Chen

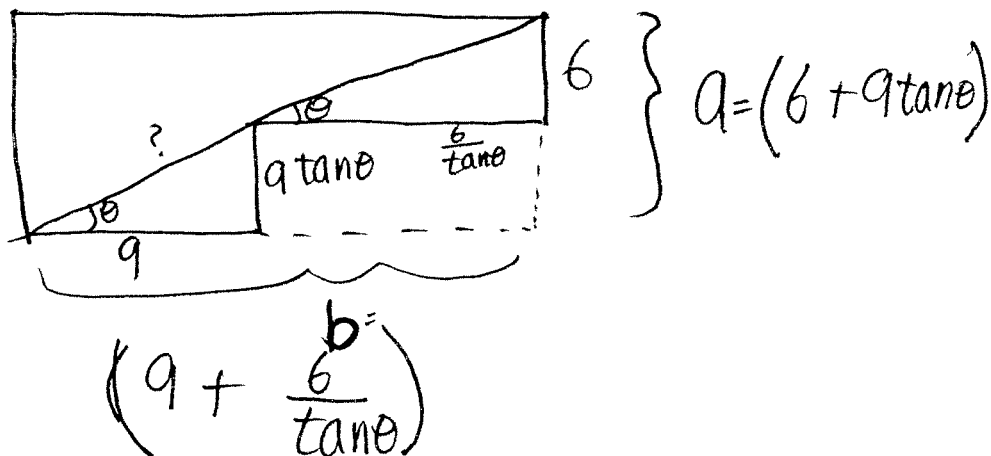
Science Buddies

#66 \nearrow (4.7)



$$\frac{6}{a} = \tan \theta$$

$$\frac{6}{\tan \theta} = a$$



Pythagorean theorem

$$a^2 + b^2 = c^2$$

$$\left(a + \frac{6}{\tan \theta}\right)^2 + (6 + a \tan \theta)^2 = c^2$$

Using calculator \Rightarrow

$$y = \left(a + \frac{6}{\tan \theta}\right)^2 + (6 + a \tan \theta)^2$$

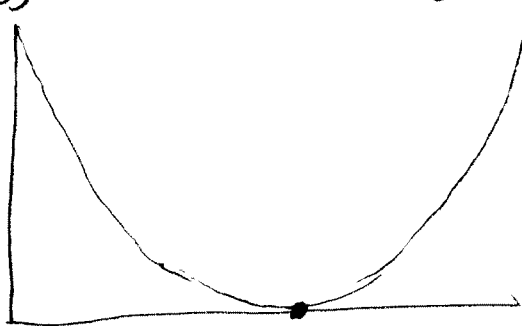
change window

$$x \text{ min} = 0$$

$$x \text{ max} = 1.5$$

then Hit

"Zoom fit"



then $\boxed{2nd} \rightarrow \text{Calc} \rightarrow \boxed{\overset{\#3}{\text{minimum}}} \text{ enter}$

left Bound = .1
enter

Right Bound = 1.4
enter = 1.388

#66

S.B

$$\text{minimum} = .38891$$

then plug minimum in $\tan \theta$

$$\sqrt{C^2} = \sqrt{\left(9 + \frac{6}{\tan(.38891)}\right)^2 + \left(6 + 9 \cdot \tan(.38891)\right)^2}$$

$C = 17.8113$ is the length of the pipe that can be carried horizontally around the corner.