L'hôpitals Rule

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\infty}{\infty} \text{ or } \frac{0}{0} \quad \text{Then} \quad \lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Ex $\displaystyle\lim_{x\to 0}\frac{\sin(x)}{x} = \frac{0}{0}$ use L'Hopitals Rule $\displaystyle = \lim_{x\to 0}\frac{\cos(x)}{1} = 1$

Ex $\displaystyle\lim_{x\to\infty}\frac{2x^2}{1x^2-7x+5} = \frac{\infty}{\infty}$ use L'HR $\displaystyle = \lim_{x\to\infty}\frac{4x}{2x-7} = \frac{\infty}{\infty} = \text{LHR} = \lim_{x\to\infty}\frac{4}{2}$

Ex $\displaystyle\lim_{x\to 0^+} x\ln x = 0\cdot\infty \Rightarrow \lim_{x\to 0^+}\frac{\ln x}{1/x} = \frac{-\infty}{\infty}$ use LHR $\displaystyle = \lim_{x\to 0^+}\frac{1/x}{-x^{-2}}$

$$= \lim_{x\to 0^+}(-x) = 0$$

Prop of Logs (Ladder)

Ex $\displaystyle e = \boxed{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x} = (1+0)^\infty \Rightarrow \lim_{x\to\infty} e^{\ln\left(1+\frac{1}{x}\right)^x} = \lim_{x\to\infty} e^{x\ln\left(1+\frac{1}{x}\right)}$

$$= e^{\lim_{x\to\infty} x\ln\left(1+\frac{1}{x}\right)} = e^{\infty\cdot 0} = e^{\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{1/x}} \quad \text{use LHR.}$$

$$= e^{\lim_{x\to\infty}\frac{\frac{1}{\left(1+\frac{1}{x}\right)}\cdot\cancel{\frac{d}{dx}\left(1+\frac{1}{x}\right)}}{\cancel{\frac{d}{dx}\left(1/x\right)}}} = e^{\lim_{x\to\infty}\frac{1}{1+\frac{1}{x}}} = e^{\frac{1}{1+0}} = e^1 = e$$

PROBLEM = $0\cdot\infty,\ \frac{0}{0},\ \frac{\infty}{\infty},\ 1^\infty,\ \infty^0,\ 0^0,\ \infty-\infty$

Ex $\displaystyle\lim_{x\to\infty}\sqrt{x}-\sqrt{x+4} = \infty-\infty = \lim_{x\to\infty}\frac{\left(\sqrt{x}-\sqrt{x+4}\right)\left(\sqrt{x}+\sqrt{x+4}\right)}{1\left(\sqrt{x}+\sqrt{x+4}\right)} = \lim_{x\to\infty}\frac{x-(x+4)}{\sqrt{x}+\sqrt{x+4}}$

↓ Algebra

$$= \lim_{x\to\infty}\frac{-4}{\sqrt{x}+\sqrt{x+4}} = 0$$