The Fundamental Theorem of CALCULUS

$$\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big|_a^b$$

Area under curve

Anti-derivative Evaluate at b & a

$$\int f(x)\,dx = F(x)+C$$

$$\int x^3\,dx = \frac{x^4}{4} + C$$

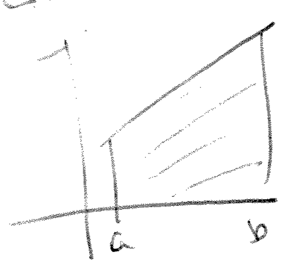
Ex $$\int_0^1 (x^2 - 3x + 5)\,dx$$

$$= \frac{x^3}{3} - \frac{3x^2}{2} + 5x \Big|_0^1$$

$$= \left[\frac{(1)^3}{3} - \frac{3(1)^2}{2} + 5(1)\right] - \left[\frac{(0)^3}{3} - \frac{3(0)^2}{2} + 5(0)\right]$$

$$= 1/3 - 3/2 + 5 = 3.8\overline{3}$$

Linear Regression



Calc 7: $\int f(x)\,dx$

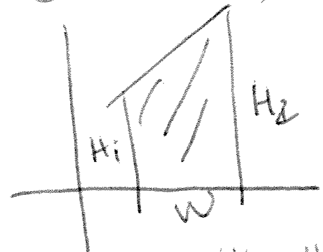
or

FTC.

$Y_2 = Ax^2 + Bx$ or

$Y_2(b) - Y_2(a)$

Geometry



$$\text{Area} = W\left(\frac{H_2 + H_1}{2}\right)$$

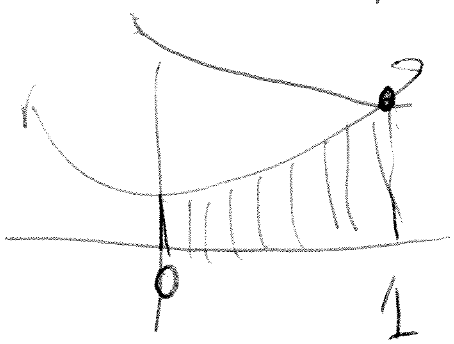
Quadrat

Add up change by Δx (not by 1) $\Delta x = \frac{1}{3}$

$$\frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \frac{4}{3} + \cdots$$

$$\sum_{i=1}^{n} \frac{i}{3} = \frac{1}{3}(1+2+3+4 \cdots = \frac{1}{3}\sum_{i=1}^{n} i$$

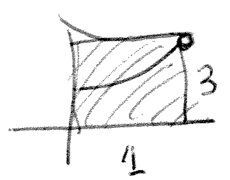
Function $y = x^2 - 3x + 5$ ← Area under between 0 and 1



$$\int_0^1 x^2 - 3x + 5 \, dx = 3.8\overline{3}$$

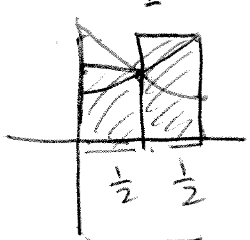
Right Side

$n = 1$



$f(1) = 3$ Area $3 \times 1 = 3$

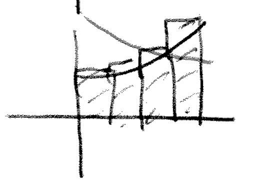
$n = 2$



$f(\frac{1}{2}) = 3.75$
$f(1) = 3$

Area $\frac{1}{2}(3.75 + 3)$

$n = 4$

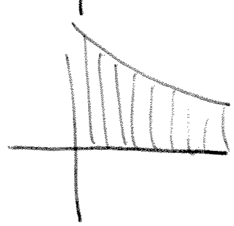


$f(\frac{1}{4}) = 4.3125$
$f(\frac{1}{2}) = 3.75$
$f(\frac{3}{4}) = 3.3125$
$f(1) = 3$

Area $\frac{1}{4}(4.3125 + 3.75 + 3.3 + 3)$

$\frac{1}{n}$ Sum(Seq(~~...~~,
(Y1, X, 1/n, 1, 1/n

$n = 100$



$\frac{1}{100}$ sum(seq(Y1, X, $\frac{1}{100}$, 1, $\frac{1}{100}$) $= 3.82$

$$\frac{1}{100} \sum_{i=1}^{100} \left(\frac{i}{100}\right)^2 - 3\left(\frac{i}{100}\right) + 5$$

$$\frac{1}{100}\left[\sum_{i=1}^{100} \frac{i^2}{100^2} - \frac{3}{100}\sum_{i=1}^{100} i + \sum_{i=1}^{100} 5\right]$$

$$\frac{1}{100}\left[\frac{1}{100^2}\sum_{i=1}^{100} i^2 - \frac{3}{100}\sum_{i=1}^{100} i + \sum_{i=1}^{100} 5\right]$$

$$\frac{1}{100}\left[\frac{1}{100^2}\left(\frac{100 \cdot 101 \cdot 201}{6}\right) - \frac{3}{100}(101)\left(\frac{100}{2}\right) + 5 \times 100\right]$$

Any

$n = \cancel{100}$

$$\frac{1}{n}\left[\frac{1}{n^2}\left(\frac{(n)(n+1)\cdot(2n+1)}{6}\right) - \frac{3}{n}(n+1)\frac{n}{2} + 5 \times n\right]$$

Algebra

$$\frac{2n^3 + \overset{\text{Lower Term}}{\text{CRAP}}}{6n^3} - \frac{3}{2}\frac{n^2 + \text{Lower Term}}{\cancel{n^2}} + 5\frac{\cancel{n}}{\cancel{n}}$$

$$\lim_{n\to\infty} \frac{2n^3 +}{6n^3} - \frac{3}{2}$$

$$\frac{1}{3} - \frac{3}{2} + 5 \quad 3.833\ldots$$

Formulas

$$\sum_{i=1}^{n} c \text{ (constant)} = \underbrace{\overset{i=1}{c} + \overset{i=2}{c} + \overset{i=3}{c} \cdots + \overset{i=n}{c}}_{n \text{ times}} = c \times n$$

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = (n+1)\left(\frac{n}{2}\right)$$

$$\sum_{i=1}^{n} i^2 = 1 + 4 + 9 + \quad + n = \frac{n(n+1)(2n+1)}{6}$$

Properties

$$\boxed{\sum_{i=1}^{n} Ai^2 + Bi + C = A\sum_{i=1}^{n} i^2 + B\sum_{i=1}^{n} i + \sum_{i=1}^{n} C}$$

Ex $$\sum_{i=1}^{5} 3i^2 + 2i - 7 = 3\sum_{i=1}^{5} i^2 + 2\sum_{i=1}^{5} i + \sum_{i=1}^{5} -7$$ $n=5$

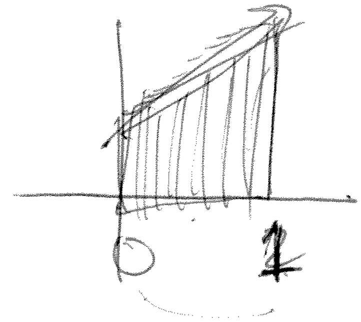
$$3 \cdot \left(\frac{5 \cdot (6)(11)}{6}\right) + 2\left((6)\left(\frac{5}{2}\right)\right) + (-7)5$$

$$= 15 \cdot 11 + 30 - 35 = \boxed{160}$$

$$\underbrace{\overset{i=1}{3(1)^2 + 2(1) - 7}} + \underbrace{\overset{i=2}{3(2)^2 + 2(2) - 7}} + \underbrace{\overset{i=3}{3(3)^2 + 2(3) - 7}}$$

Prop $$\sum_{i=50}^{1000} i^2 = \sum_{i=1}^{1000} i^2 - \sum_{i=1}^{49} i^2$$

$f(x) = 4x + 5$



$\Delta x = \frac{1}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} 4\left(\frac{i}{n}\right) + 5 = 4 \sum_{i=1}^{n} \left(\frac{i}{n}\right) + \sum_{i=1}^{n} 5$$

$n = \#$ of rectangles

$$\cancel{\lim_{n\to\infty} f(n) = \text{Area under curve}}$$

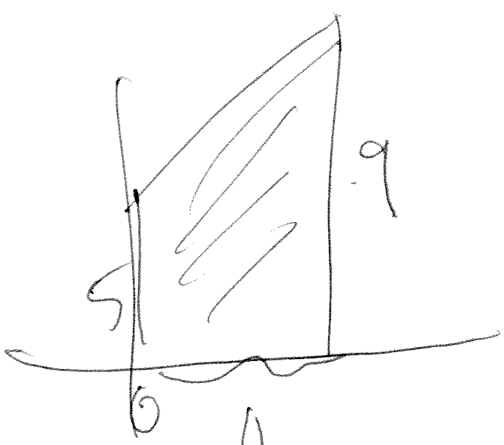
$$= \frac{4}{n} \sum_{i=1}^{n} i + \sum_{i=1}^{n} 5$$

$$= \left(\frac{4}{n}(n+1)\left(\frac{n}{2}\right) + 5n\right)\Delta x$$

$$= \left(\frac{\cancel{4}}{\cancel{n}}(n+1)\left(\frac{\cancel{n}}{2}\right) + 5n\right)\frac{1}{n}$$

$$= \frac{2n+1+5n}{n} = \frac{7n+1}{n}$$

$$\lim_{n\to\infty} \frac{7n+1}{n} = \frac{7}{1}$$



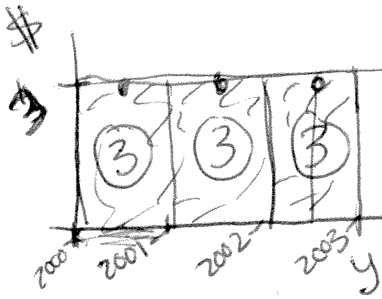
$\frac{9+5}{1} = \frac{14}{2} = 7$

Anti Derivative $\int f(x)\,dx = F(x) + c$

Indefinite Integral

Area Under $f(x)$ between a & b $\int_a^b f(x)\,dx = \text{Value}$

Definite Integral



Year

$f(x) = 3$

$\int_{2000}^{2003} 3\,dx$ = Total Sales

$= 9$

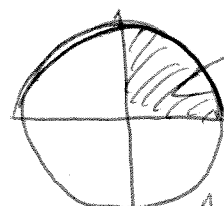
Adding Rectangles

height x base

$f(2000)(1yr) + f(2001)(1yr) + f(2002)(1yr) = 9$

Estimate

$\int_0^2 \sqrt{4-x^2}\,dx$

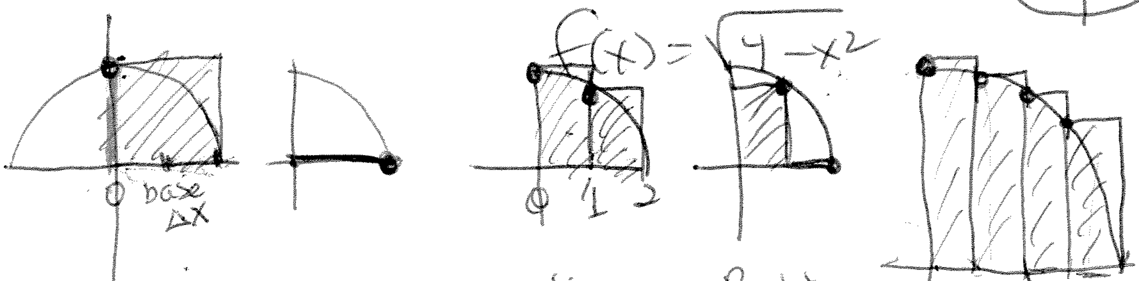


$\pi = 3.14$

Area Circle $(r=2)$

4π

$f(x) = \sqrt{4-x^2}$



Left End Point 1 rectangle	Right END 1 rect	Left END 2 Rect	Right END 2 Rect	Left END
$f(0)(2-0)$	$f(2)(2-0)$	$f(0)(1-0)+f(1)(2-1)$	$f(1)\cdot(1-0)$	$\frac{1}{2}[f(0)+f(\frac{1}{2})+f(1)+f(1.5)]$
$2\cdot 2$	$0\cdot 2$	$2(1)+\sqrt{3}\cdot(1)$	$\sqrt{3}$	$\frac{1}{2}[2+\sqrt{3.75}+\sqrt{3}+\sqrt{4-(1.5)^2}]$
4	0	$2+\sqrt{3}$	1.73	
		3.73		

Right End

$\frac{1}{2}[f(\frac{1}{2})+f(1)+f(1.5)+f(2)]$

To Add a "List" List = $\boxed{2^{nd}}\boxed{STAT}$

LIST >> Math 5: Sum(

LIST > OPS 5: Seq(

$$\text{Sum}(\text{seq}(\cancel{Y_1}, X, \underset{\text{LOWER}}{\underline{0}}, \underset{\text{UPPER}}{\underline{1.5}}, \underset{\Delta X}{\underline{.5}})) * .5$$

Left → 3.495...

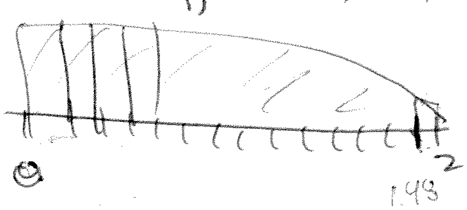
Edit Sum(seq(Y1, X, .5, 2, .5)) * .5

Right → 2.495...

Add 100 rectangles! WOW!

$$\text{Sum}(\text{seq}(Y_1, X, 0, \overset{\overset{b-\Delta X}{2-.02}}{1.98}, \overset{\Delta X}{.02})) * \overset{\Delta X}{.02} = 3.1604$$

Left

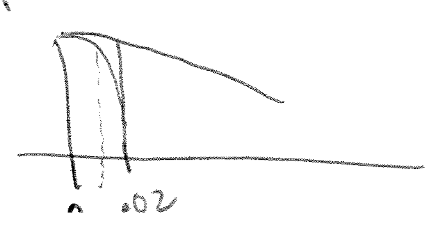


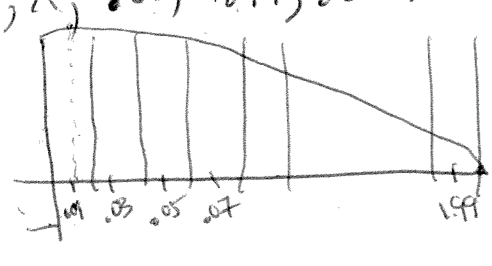
$$\frac{2-0}{100} = \frac{1}{50} = .02$$

Want $\lim_{N\to\infty} A(N)$ = Exact Area

Right $\text{Sum}(\text{Seq}(\overset{\text{Heights}}{Y_1, X, .02, 2, .02}) * \overset{\text{base}}{.02})$

Midpoint $\text{Sum}(\text{Seq}(Y_1, X, .01, 1.99, .02) * .02)$





Calc 7: $\int f(x)$

$n = 100$ Right

Sum(Seq(Y1, X, .1, 10, .1))*.1

$n = 1000$

Sum(Seq(Y1, X, .1, 10, .01))*.01

Answer Wrote

Under
Left

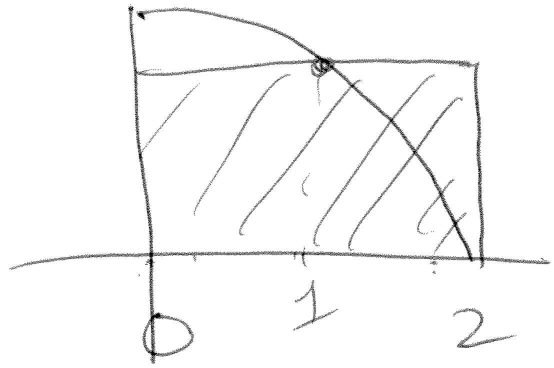
(Y1, X, 0, 9.9, .1)*.1

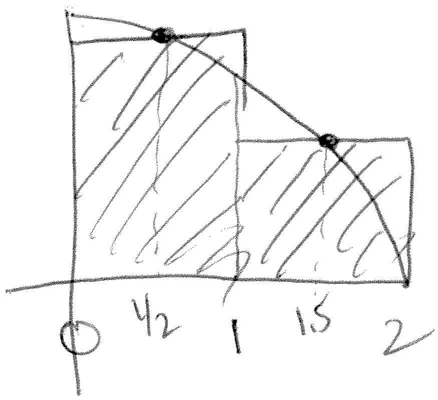
Sum(Seq(Y1, X, start, finish, Δx))*

$$\Delta x = \frac{b-a}{n} \qquad \frac{10-0}{100} = \frac{10}{100} = .1$$

$$\frac{2010 - 2008}{1000} =$$

Midpoint





Midpoint
1 rectangle

Summation Sigma Notation

$$\sum_{i=a}^{b} f(i) = f(a) + f(a+1) + f(a+2) + f(a+3) \dots + f(b)$$

add them up

index Sigma

$$\sum_{i=1}^{3} i^2 = 1^2 + 2^2 + 3^2 = 14$$

$i=1$ $i=2$ $i=3$

$$\sum_{i=1}^{100} i = 1 + 2 + 3 + \dots + 98 + 99 + 100 = (100+1)\left(\frac{100}{2}\right)$$

$= 101 \cdot 50 = 5050$

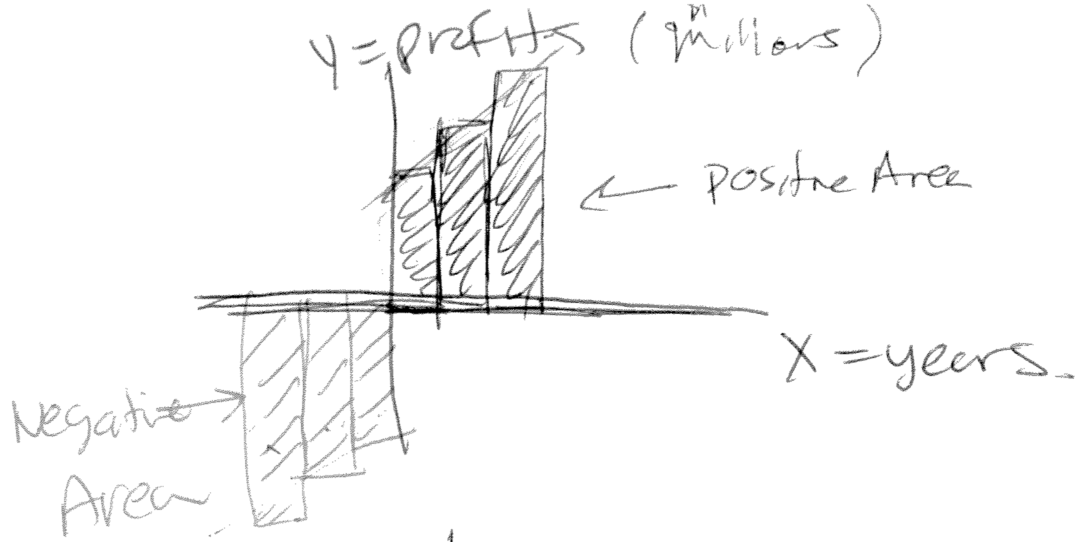
$$\boxed{\sum_{i=1}^{n} i = (n+1)\left(\frac{n}{2}\right)}$$

100 rectangles
$\Delta = .02$

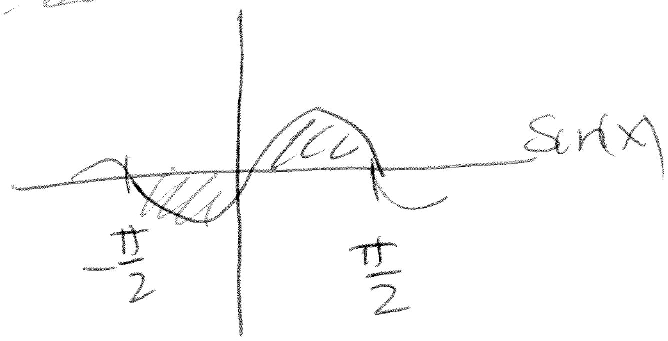
$$\sum_{n=1}^{100} \sqrt{4 - \left(\frac{n}{50}\right)^2}$$

$$\sum_{i=1}^{3} 8 = 8 + 8 + 8 = 24$$

$i=1$



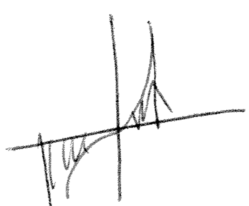
x	y
1	3
2	4
3	5



$$\int_{-\pi/2}^{\pi/2} \sin(x)\,dx = 0$$

$$\int_{-a}^{a} \text{odd function} = 0$$

$$\int_{-a}^{a} \sin(x)\,dx = 0$$



$Y_1 = x + 2$

Calc 7: $\int f(x)dx$

Lower: 0

Upper: 3

$\int f(x)dx = 10.5$

$Y_1 = \sqrt{(4 - x^2)}$

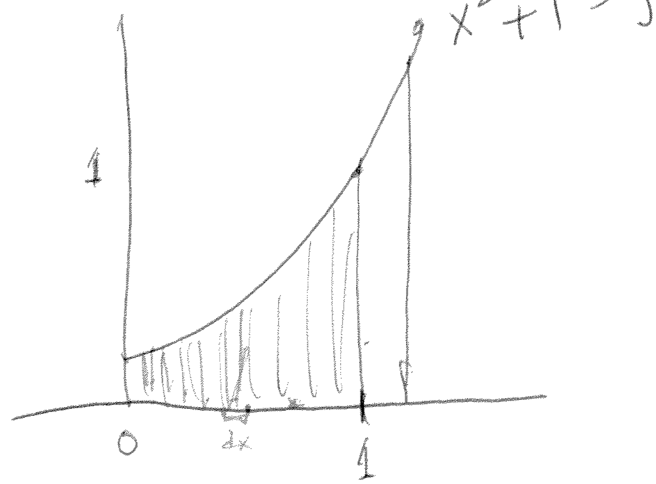
Calc 7: $\int f(x)dx$

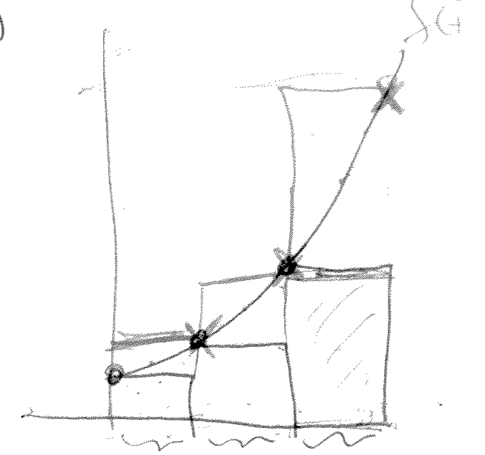
Lower: -2

Upper: 2

$\int f(x)dx = 6.28\ldots$

$= 2\pi$





$A_1=2$ $A_2=3$ $A_3=6$

Area = 2+3+6
= 11

$A_1=4+6+14$
= 24

$$\text{AREA} = \int_a^b f(x)\,dx$$

definite Integral

Rectangles

Limit to make $\Delta x \to dx$

$$\int_a^b f(x)\,dx$$

Area under f(x) between a & b

$$F(x) = \int f(x)\,dx$$

indefinite Integral

Anti-derivative

$$F'(x) = f(x)$$