Calculus = Study of Change (Functions)

$\left\{\begin{matrix}\text{Increase}\\ \text{Decrease}\\ \text{Constant}\end{matrix}\right\}$ Instantaneous

$\left[\begin{matrix}\text{concave up}\\ \text{concave down}\end{matrix}\right]$

Average $\longrightarrow \dfrac{\Delta y}{\Delta x}$

"$\lim\limits_{\Delta x \to 0}$" $\dfrac{\Delta y}{\Delta x}$

Invent Limits

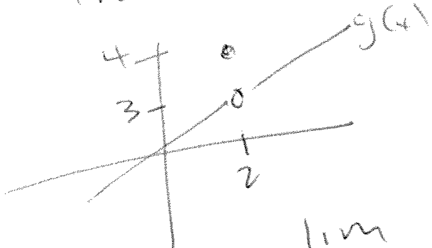
Usually. $\lim\limits_{x \to a} f(x) = f(a)$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\cancel{h}(f'(x)+h)}{\cancel{h}} = f'(x)$$

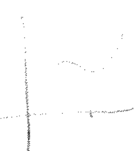
ex $$\lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h} = \cos(x)$$

$\boxed{\varepsilon - \delta \text{ definition}}$

Limit from Graphs + Data

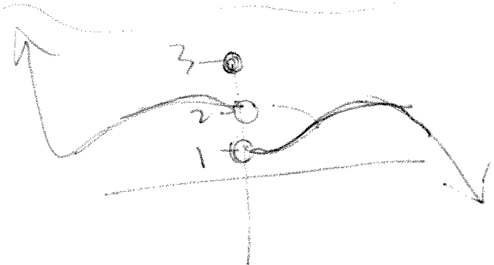


x	y
1.9	2.9
1.99	2.99
2.001	3.01
2.1	3.1



$\lim\limits_{x \to 2} g(x) = 3$ $\qquad \lim\limits_{x \to 2} y = 3$

$$\lim_{h \to 0} \frac{\sin(h)}{h} = \lim_{h \to 0} \frac{\cos(h)}{1} = 1$$



$\lim\limits_{x \to 0^+} f(x) = 1$ $\qquad \lim\limits_{x \to \infty} f(x) = -\infty$

$\lim\limits_{x \to 0^-} f(x) = 2$ $\qquad f(0) = 3$

$\lim\limits_{x \to 0} f(x) = \text{DNE}$ $\qquad \lim\limits_{x \to -\infty} f(x) = \infty$

Derivative = Slope of Tangent Line

$= \frac{dy}{dx}$ = instantaneous Rate of Change

Definition $= \boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

Power

$$\frac{d}{dx} x^n = n x^{n-1}$$

Chain Rule

$$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot \frac{dg}{dx}$$

Product

$$\frac{d}{dx} f \cdot g = f'g + g'f$$

Quotient

$$\frac{d}{dx} \frac{f}{g} = \frac{gf' - fg'}{g^2}$$

$$\frac{d}{dx} f(u) = f'(u) \cdot u'$$

Implicit $\frac{d}{dx}$ both sides

Logarithmic diff. x^x

Differentials $dy = f'(x)dx$

errors.

Related Rate $\frac{d}{dt}$

MAX / MINS

Concavity f''

Position	Velocity	Acceleration
$S(t)$	$v(t) = s'(t)$	$a(t) = v'(t) = s''(t)$

Anti Derivative or Integral (indefinte)

$F(x)$ $\int f(x)\,dx$

Ex $V(t) = \frac{1}{2}t + 10$, and $S(0) = 100$
INITIAL VALUE

$V(0) = 10$
$V(10) = 15$
$V(20) = 20$

what $a(t) = V'(t) = \frac{1}{2}$

what $S(t) = \int V(t)\,dt$

$= \int \frac{1}{2}t + 10 \, dt$

$S(t) = \frac{t^2}{4} + 10t + c$

use $S(0) = 100$

$S(0) = \frac{0}{4} + 10 \cdot 0 + c$
$= 10$

$C = 100$

$S(t) = \frac{t^2}{4} + 10t + 100$

Ex $V(t) = e^t + \sqrt{t}$ $S(1) = 2$

$e^t + t^{1/2}$

$\int V(t)\,dt = \int e^t + t^{1/2}\,dt$

$S(t) = \left[e^t + \frac{t^{3/2}}{3/2} + c\right]$

$S(1) = e^1 + \frac{2}{3} + c = 2$

$\Rightarrow$ $C = 2 - 2/3 - e$

$S(t) = e^t + \frac{2}{3}t^{3/2} + 1\frac{1}{3} - e$

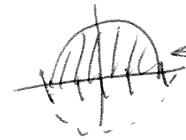
Definite Integral

$$\int_a^b f(x)\,dx \equiv$$ Area under curve $f(x)$ from $x=a$ $x=b$

Geometry

$$\int_{-2}^{2} \sqrt{4-x^2}\,dx$$

$A = \pi(2)^2 = 4\pi$



$\frac{4\pi}{2} = 2\pi$

$y = \sqrt{4-x^2}$

$y^2 = 4 - x^2$

$x^2 + y^2 = 4$

$Y_1 = \sqrt{(4-x^2)}$

Calculator

$Y_1 = \sqrt{(4-x^2)}$

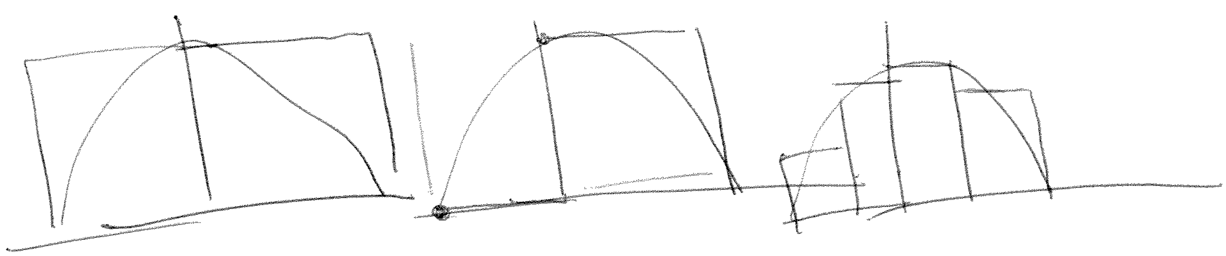
Calc 7: $\int f(x)dx$

Lower: -2

Upper: 2

$\int f(x)dx = 6.28...$

Rectangular Approx



Sum(seq(Y_1, X, -2, 2, .01) $\times$ .01

FTC

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(b)}$$

Techniques of Integration

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \quad \text{Power Rule}$$

$$\int e^x\,dx = e^x + c \qquad \int \sin x\,dx = -\cos x + c \qquad \int \cosh(x)\,dx = \sinh(x) + c$$

Lots of . . .

$$\int f'(u)\,du = f(u) + c$$

Ex

$$\int e^{x^2}\cdot \underbrace{2x\,dx} \qquad u = x^2 \qquad \underbrace{du} = \underbrace{2x\,dx}$$

$$\boxed{\int e^u\,du = e^u + c}$$

$$= e^{x^2} + c$$

Ex

$$\int \frac{2x+5}{x^2+5x+7}\,dx \qquad u = x^2+5x+7 \qquad du = 2x+5\,dx$$

$$\boxed{\int \frac{du}{u} = \ln|u| + c}$$

$$= \ln|x^2+5x+7| + c$$

Ex $\int x \sin(x^2)\,dx$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$\boxed{\begin{aligned}&\int \sin(u)\frac{du}{2}\\&\frac{1}{2}\int \sin(u)\,du\\&-\frac{1}{2}\cos(u) + C\end{aligned}}$$

$-\frac{1}{2}\cos(x^2) + C$

Ex $\int e^{2x^3}\sin(e^{2x^3})x^2\,dx$

$u = e^{2x^3}$

$\frac{du}{6} = \frac{e^{2x^3}\cdot 6x^2\,dx}{6}$

$$\boxed{\begin{aligned}&\int \sin(u)\frac{du}{6}\\&\frac{1}{6}\int \sin(u)\,du\\&-\frac{1}{6}\cos(u) + C\end{aligned}}$$

$-\frac{1}{6}\cos\left(e^{2x^3}\right) + C$

$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$

$\int \frac{x}{1+x^2}\,dx = \int \frac{1}{u}\frac{du}{2}$

$= \frac{1}{2}\ln|u|$

$= \frac{1}{2}\ln|1+x^2| + C$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\int \frac{x^2}{1+x^2}\,dx$

$$x^2+1 \overline{\big) x^2+0x+0} \quad 1 - \frac{1}{x^2+1}$$

$$1+x^2 \overline{\big) x^2+0x+0}$$

$$\begin{array}{r} x^2+0x+0 \\ -(x^2 \quad\; +1) \\ \hline -1 \end{array}$$

$$\int \frac{x^2}{1+x^2}\,dx = \int 1 - \frac{1}{x^2+1}\,dx$$

$$= \int dx - \int \frac{1}{x^2+1}\,dx$$

$$= x - \tan^{-1}(x) + C$$

Ex

$$\int x\sqrt{4-x^2}\,dx$$

$u = 4-x^2$

$du = -2x\,dx$

$-\frac{du}{2} = x\,dx$

$$= \int u^{1/2}\,\frac{du}{-2}$$

$$= \frac{u^{3/2}}{-3} + C$$

$$= -\frac{1}{3}(4-x^2)^{3/2} + C$$

Ex $\int x^2 \sqrt{4-x^2}\, dx \Rightarrow$

$u = 4 - x^2$

$x^2 = 4 - u$

$dx = -du$

$\int (4-u)\sqrt{u}\,(-du)$

$\int -4\sqrt{u} + u\sqrt{u}\, du$

$\int -4u^{1/2} + u^{3/2}\, du$

$\dfrac{-4u^{3/2}}{3/2} + \dfrac{u^{5/2}}{5/2} + C$

$=$

$$\int_a^b f'(u)\,du = f(u(b)) - f(u(a)) = \cancel{f(u(x))}\Big|$$

Ex $\displaystyle\int_{x=1}^{2} x e^{x^2}\, dx$

$u = x^2 \quad du = 2x\,dx$

$\dfrac{du}{2} = x\,dx$

$$= \int_{u=1}^{4} e^u \cdot \frac{du}{2} = \frac{1}{2} e^u \Big|_1^4$$

$$= \frac{1}{2}\left(e^4 - e^1\right)$$