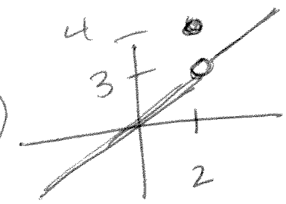
CALCULUS = Study of Change

{ Increase, Decrease, Constant } X { concave up, concave down, Inflection Pt. } { Average or Instantaneous } Rate of change.

Ave ⇒ 2 points $\quad \dfrac{\Delta y}{\Delta x} \quad$ Instant. ⇒ 1pt

Limit

Normal $\quad \lim\limits_{x \to a} f(x) = f(a)$



GRAPH

x	y
1.9	2.9
1.99	2.99
2.01	3.01
2.1	3.1

TABLE

Common Factor + simplify

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{0}{0}$$

$$\lim_{h \to 0} \frac{\cancel{h}\,(g(x,h))}{\cancel{h}} = f'(x)$$

Use L'Hopitals

$$\lim_{h \to 0} \frac{\sin(h)}{h} = \cancel{\frac{0}{0}} = \lim_{h \to 0} \frac{\cos(h)}{1} = 1$$

END Behavior or Squeeze Thrm

$$\lim_{x \to \infty} \frac{\sin(x)}{x} = \frac{\text{UND}}{\infty}$$

$$\frac{-1}{x} \le \frac{\sin(x)}{x} \le \frac{1}{x}$$

$$\lim_{x \to \infty} \frac{-1}{x} \le \boxed{\lim_{x \to \infty} \frac{\sin x}{x}} \le \lim_{x \to \infty} \frac{1}{x}$$

$$0 = \qquad\qquad = 0$$

$= 0$

Derivative

Definition $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

Y_1 = functn
calc 6: dy/dx

Power $\frac{d}{dx} x^n = n x^{n-1}$

Product $\frac{d}{dx} f \cdot g = fg' + gf'$

Quotient $\frac{d}{dx} \frac{f}{g} = \frac{gf' - fg'}{g^2}$

$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx} \ln x = \frac{1}{x} \quad x > 0$

$\frac{d}{dx} \sinh(x) = \cosh(x)$

$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$

$\boxed{\text{Chain}}$ $\frac{d}{dx}(f \circ g) = \frac{d}{dx} f(g(x)) \Rightarrow f'(\underbrace{g(x)}_{\text{CRAP}}) \cdot g'(x)$

Ex $\frac{d}{dx} \sin(x^2 + 3x) = \cos(x^2+3x) \cdot \frac{d}{dx}(x^2+3x) = \cos(x^2+3x) \cdot (2x+3$

$x^{\cos}$

Implicit ($\frac{d}{dx}$ of both sides)

Logarithmic (Differen) (**ln** of both sides first, Then d/dx of both)

Related Rates (balloon) (d/dt of both sides)

Differentials ($dy = f'(x)\,dx$) ERROR

MAX/MIN $dy/dx = 0$ critical Points

Concavity $y'' > 0$ up (MIN) $y'' < 0$ down (MAX)

Linearizing (Find $\sqrt{10}$)

Newton's method [Finding zeros]

$\# \to X$

$X - Y_1 / \text{nDeriv}(Y_1, X, X) \to X$

$=$

$=$

Anti-Derivatives Indefinite Integral

$F(x) = \int f(x)\,dx \Rightarrow$

Power:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

Ex

$$\int \sqrt{x}\,(x^{3/2} + 7)\,dx = \int x^2 + 7x^{1/2}\,dx$$

$$= \frac{x^3}{3} + \frac{7x^{3/2}}{3/2} + C$$

Ex

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$$

Ex

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$$

Ex

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$x \neq 0$

New

$$\int \sin(x^2+3x) \cdot (2x+3)\,dx$$

CRAP d(CRAP)

$\boxed{\text{Substitution}}$

$u = x^2 + 3x$

$du = (2x+3)\,dx$

$$\int \sin(u) \cdot du \Rightarrow -\cos(u) + C$$

$$\Rightarrow -\cos(x^2+3x) + C$$

Ex $\int \cosh(e^x) \cdot e^x \, dx = \sinh(e^x) + c$

Substitution

$u = e^x$

$du = e^x \, dx$

$\int \cosh(u) \cdot du = \sinh(u) + c$

Ex $\int x e^{\sin(x^2)} \cdot \cos(x^2) \, dx = \frac{1}{2} e^{\boxed{\sin(x^2)}} + c$

Substitute $u = \sin(x^2)$

$\frac{du}{2} = \cos(x^2) \cdot \frac{\cancel{2} x}{\cancel{2}} \, dx$

$\int e^u \frac{du}{2} = \frac{1}{2} \int e^u \, du = \frac{1}{2} e^{\boxed{u}} + c$

Ex $\int \frac{x^2+2}{x^3+6x} \, dx = \frac{1}{3} \ln |x^3+6x| + c$

Substitute

$u = x^3 + 6x$

$du = (3x^2 + 6) \, dx$

$\frac{du}{3} = \frac{\cancel{3}}{\cancel{3}}(x^2+2) \, dx$

$\int \frac{1}{u} \frac{du}{3} = \frac{1}{3} \int \frac{1}{u} du = \frac{1}{3} \ln |u| + c$

Ex

$$\int \tan x \, dx = \int \frac{1 \sin x}{\boxed{\cos x}} dx = -\ln|\cos x| + c$$

Substitute

$u = \cos x$

$du = -\sin x \, dx$

$(-1) \quad (-1)$

$$\int \frac{1}{\boxed{u}} \, -du = -\ln|u| + c$$

$$-\int \frac{1}{u} du = -\ln|u| + c$$

$$\int du = u \qquad \int \frac{1}{u} du = \ln|u|$$

$$\boxed{\frac{d}{du} \ln|u|} = \boxed{\frac{1}{u} du}$$

$$\frac{d}{dx} x^2 = 2x$$

$$\frac{d}{dt} x^2 = 2x \frac{dx}{dt}$$

$$d(x^2) = 2x \, dx$$

$$d(\ln u) = \frac{1}{u} du$$

$$\int \frac{-du}{u} = \int -\frac{1}{u} du$$

Ex

$$\int x\sqrt{x-4}\,dx = \frac{(x-4)^{5/2}}{5/2} + \frac{4(x-4)^{3/2}}{3/2} + C$$

Substitution

$u = x-4$ so $x = u+4$

$du = dx$

$$\int (u+4)\sqrt{u}\,du = \int u^{3/2} + 4u^{1/2}\,du$$

$(u+4)u^{1/2}$

$$= \frac{u^{5/2}}{5/2} + \frac{4u^{3/2}}{3/2} + C$$

Application of Antiderivation

$s(t)$ = position

$v(t)$ = velocity $= s'(t)$

$a(t)$ = acceleration $= s''(t) = v'(t)$

Initial Value Problems

$v(t) = 2t + 11$ $\quad s(1) = 3$ $\quad$ Find $s(t) = ?$

$s'(t) = 2t + 11$

$s(t) = \int (2t+11)\,dt$

$s(t) = t^2 + 11t + C$

$s(1) = 1 + 11 + C = 3$

$C = 3 - 12 = -9$

$s(t) = t^2 + 11t - 9$

Ex $\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$

Ex $\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\ln|1+x^2| + C$

Ex $\int \frac{x^2}{1+x^2}\,dx = x - \tan^{-1}(x) + C$

$$\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\ln|1+x^2| + C$$

$$u = 1+x^2$$

$$\frac{du}{2} = \frac{2x\,dx}{2}$$

$$\int \frac{1}{u}\,\frac{du}{2} = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C$$

$$\begin{array}{r|l} & 1 - \frac{1}{x^2+1} \\ \cline{2-2} x^2+1 & x^2+0x+0 \\ & -(x^2 \qquad +1) \\ \cline{2-2} \end{array}$$

$$\int \frac{x^2}{1+x^2}\,dx = \int 1 - \frac{1}{x^2+1}\,dx = x - \tan^{-1}(x) + C$$

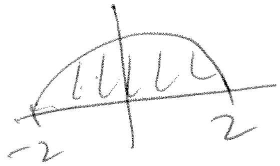
$$= \int 1\,dx - \int \frac{1}{x^2+1}\,dx$$

Definite Integrals

$$\int_a^b f(x)\,dx$$

→ Area under $f(x)$ between a & b

Geometry

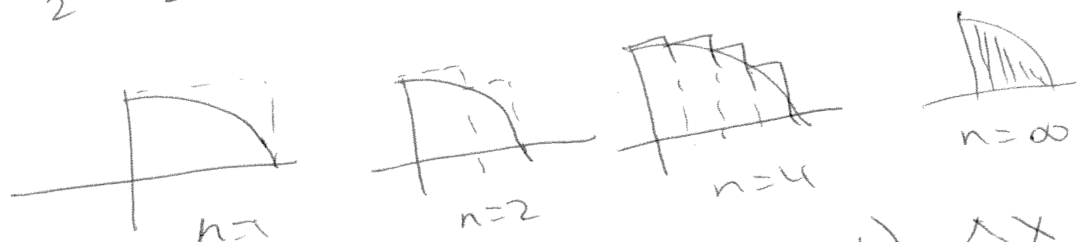


$$\int_{-2}^{2} \sqrt{4-x^2}\,dx = 2\pi$$

Calc 7: $\int f(x)dx$
Lower -2
Upper 2
6.28

$$\frac{A}{2} = \frac{\pi r^2}{2} = 2\pi$$

Rectangles



Sum(seq(Y_1 , X, Lower, Upper, ΔX)) · ΔX

Area

Fundamental Theorem of Calc

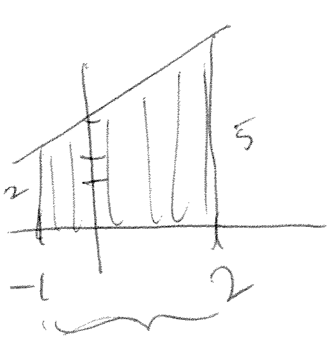
$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Ex

$$\int_{-1}^{2} x+3\,dx = \left.\frac{x^2}{2} + 3x\right|_{-1}^{2}$$

$$= \left(\frac{4}{2} + 6\right) - \left(\frac{1}{2} - 3\right)$$

$$= 8 + 2\tfrac{1}{2} = 10.5$$



$\frac{1}{2}(2+5)\cdot 3$

$\frac{7}{2}\cdot 3 = \frac{21}{2} = 10.5$

F.T.C with Substitution

Ex $\displaystyle\int_{x=1}^{2} \sin(x^2+2)\, 2x\,dx$ = X

Substitute $u = x^2+2$

$du = 2x\,dx$

$u(2) = 2^2+2 = 6$

$u(1) = 1^2+2 = 3$

$$\int_{3}^{6} \sin(u)\,du = -\cos(u)\Big|_3^6$$

$$= -\cos 6 + \cos(3)$$

Ex e^{3x}