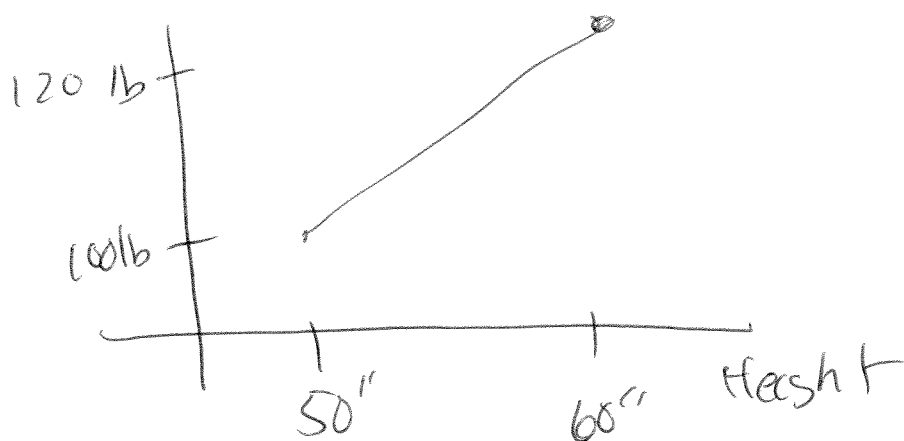


CALCULUS = study of change (of Functions)

3 ways of change  $\left\{ \begin{array}{l} \text{Increase} \\ \text{Decrease} \\ \text{Constant.} \end{array} \right.$

2 types of change  $\left\{ \begin{array}{l} \text{Average (2 pts)} \\ \text{Instantaneous (1 pt)} \end{array} \right.$

Find Average Rate of Change.



$$\bar{m} = \frac{\Delta y}{\Delta x} = \frac{120 - 100}{60 - 50} = \frac{20}{10} = 2 \text{ lbs/in.}$$

# Instantaneous Rate of Change

calc = 2nd trace 6:  $dy/dx$

$$x = 100 \quad dy/dx = -23.67 \dots$$

AT 1 POINT

$$f(x) = x^2 - 7$$

Average Rate of change between 2 & 3

$$f(3) = 9 - 7 = 2$$

$$f(2) = 4 - 7 = -3$$

x	y
2	-3
3	2

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2 - (-3)}{3 - 2} = 1$$

More general

$$f(2+h) = (2+h)^2 - 7$$

$$f(2) = -3$$

x	y
2	-3
2+h	-3+4h+h <sup>2</sup>

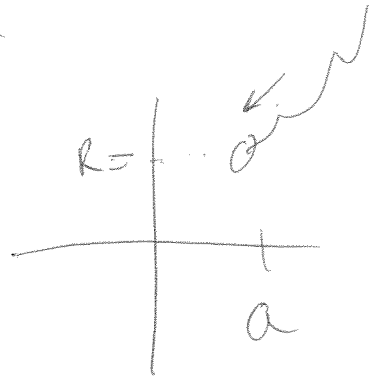
$$4 + 4h + h^2 - 7$$

$$m = \frac{\Delta y}{\Delta x} = \frac{4h + h^2}{h} = \frac{h}{h} \cdot (4 + h)$$

# Two Sided Limits

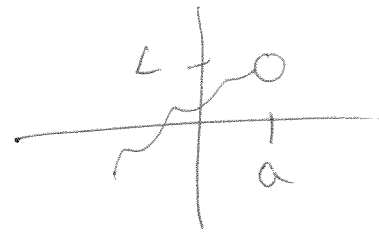
Right Sided Limit

$$\lim_{x \rightarrow a^+} f(x) = R$$



Left Sided Limit

$$\lim_{x \rightarrow a^-} f(x)$$



$$\lim_{x \rightarrow 150^+} r(x) = 2570.3$$

$$\lim_{x \rightarrow 150^-} r(x) = 3366.7$$

Given  $f(x) = x^2$

Find 
$$\frac{f(x+h) - f(x)}{h}$$

Instantaneous Rate

Let "h" be 0

---

$$y = \frac{x^2 - 4}{x - 2} = \underline{x + 2}$$

$$\text{At } x = 2 \quad y = 4$$

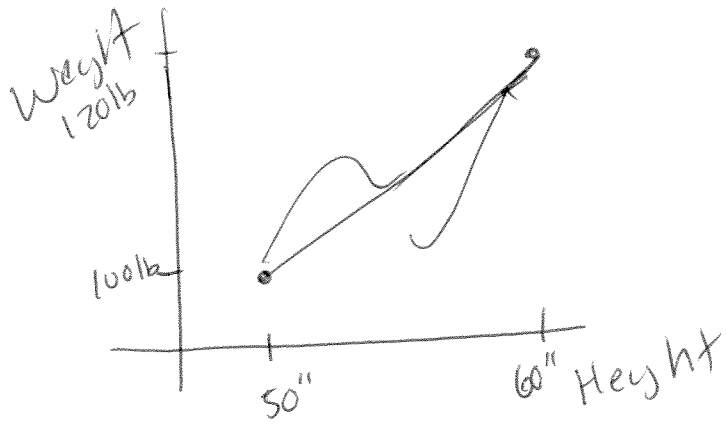
$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$$

---

$$\lim_{x \rightarrow 3} \frac{x^2 - 4}{x - 2} = \frac{3^2 - 4}{3 - 2} = 5$$

# LIMIT

Average Rate of Change



$$\bar{m} = \frac{\Delta y}{\Delta x} = \frac{120 - 100}{60 - 50} = \frac{20}{10} = 20 \text{ lb/in}$$

50	5734.4
300	<del>5734.4</del> 1050.

$$= \frac{46}{-250}$$

STAT → CALC 4: Linr

Y= VARS 5: 1:

STAT → CALC 5: Quad

Y= Y<sub>2</sub> VARS 5: 1:

split in middle X = 150

$$Y_1 = \text{Linreg} / (X \leq 150)$$

↑  
2nd Math 6: ≤  
4: ≥

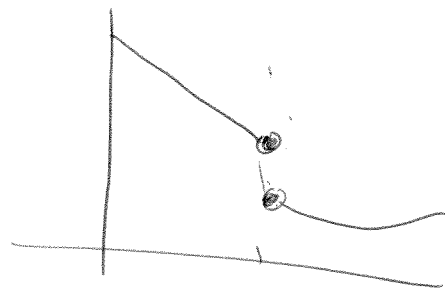
$$Y_2 = \text{Quadreg} / (X \geq 150)$$

Evaluate at multiple values

tblset = 2nd Window

✓ ✓ → Ask Center →

Table = 2nd Graph



$$f(50) = 5734.4$$

$$f(100) = 4550.5$$

$$f(200) = 1050$$

$$f(150) = \left\{ \begin{array}{l} 3366.7 \\ \text{or} \\ 2578.3 \end{array} \right\}$$

$$f(x) = \left\{ \begin{array}{ll} \text{linreg} & x \leq 150 \\ \text{quad} & x \geq 150 \end{array} \right.$$

GROUP NAME: Mathletes

Student Names (First and Last)

Logo:



Speaker/Presenter: Kyle Inverso

Date: 8/28/13

Writer/Prep: Aidan Callahan

Topics: rate of change

QC/Leader: Loogan Hockenbury

Instructions:

X	Y
1927	100
2007	30

$$m = \frac{\Delta y}{\Delta x} = \frac{100 - 30}{2007 - 1927} = \frac{7}{8} \frac{\text{grade}}{\text{year}} = .875 \frac{\text{grade}}{\text{year}}$$

in 1957, the instantaneous rate of change was

$$-.0092 \frac{\text{grade}}{\text{year}}$$

X	Y
1927	100
1937	92
1947	85
* 1957	72 *
1967	62
1977	51
1987	42
1997	37
2007	30

GROUP NAME: <u>The Wolfpack</u>	Student Names (First and Last)
Logo:	Speaker/Presenter: <u>Jared S.</u>
Date: <u>8/28/13</u>	Writer/Prep: <u>Dominic Connor (DC)</u>
Topics: <u>Linear</u>	QC/Leader: <u>Quayshawn J.</u>

Instructions:

YEAR	SPEED
1996	14k
1998	28k
2001	56k
2003	5m
2006	10m

$$m = \frac{\Delta y}{\Delta x} = \frac{10000 - 1400}{2006 - 1996}$$


$$= \frac{998.6}{10}$$

$$m = 998.6 \text{ kb/year}$$

$$x = 2004 \quad dy/dx = 4248.7671 \text{ kb/year}$$

$$x = 2000 \quad dy/dx =$$



GROUP NAME: <u>Irish Math Bombs</u>	Student Names (First and Last)
Logo: 	Speaker/Presenter: <u>Bobby O'Connor</u>
Date: <u>8/28/13</u>	Writer/Prep: <u>Connor Krugman</u>
Topics:	QC/Leader: <u>William Smith</u>

Instructions:  
Average Rate of Change

FBR Price over the last 10 years

ARC:  $\frac{\Delta y}{\Delta x}$   ~~$\frac{y_2 - y_1}{x_2 - x_1}$~~

ASK calc

$\frac{\Delta y}{\Delta x}$	$\frac{x}{y}$
	$\frac{3}{13.05}$
	$\frac{13}{20.98}$

<del>2000</del>	<del>13</del>
<del>2001</del>	<del>15</del>
<del>2002</del>	<del>16</del>
<del>2003</del>	<del>19</del>
<del>2004</del>	<del>20</del>
<del>2005</del>	<del>21</del>
<del>2006</del>	<del>20</del>
<del>2007</del>	<del>19</del>
<del>2008</del>	<del>16</del>
<del>2009</del>	<del>15</del>
<del>2010</del>	<del>13</del>
<del>2011</del>	<del>13</del>
<del>2012</del>	<del>13</del>
<del>2013</del>	<del>13</del>

$\frac{7.93}{10} = .793$  dollars per year

79¢ rise per year

year 2000's	Price
x	y
3	13
5	15
7	16
9	19
11	20
13	21

Calculating Instantaneous Rate of Change

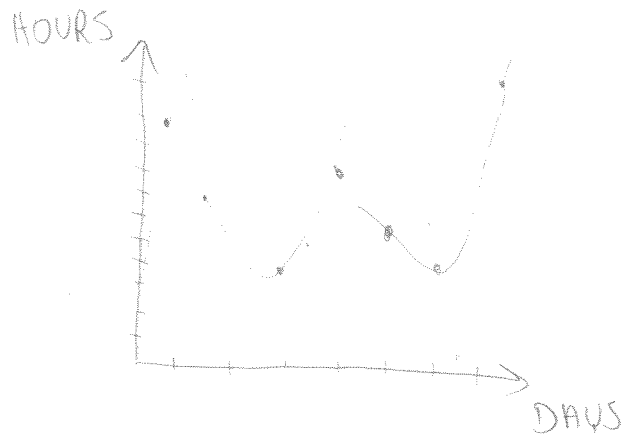
in 2005, the r.o.c was ~~83¢~~  
 .836  
84¢ per year

GROUP NAME: Apples 2 Apples	Student Names (First and Last)
Logo:	Speaker/Presenter: Thomas Y
Date: 02/28/13	Writer/Prep: ANNA S
Topics:	QC/Leader: Steve H

Instructions:

Average Rate of Change

DAY	HOURS
Sunday	10
Monday	7
Tuesday	4
Wednesday	9
Thursday	6.5
Friday	4.5
Saturday	11



SIN reg for  $x \leq 4$   
 quartreg for  $x \geq 4$

Average Rate of Change

$$\bar{m} = \frac{\Delta y}{\Delta x} = \frac{11-10}{7} = \frac{1}{7}$$

Instantaneous Rate of Change:

On Monday  $dy/dx = -7.36$  hours/day

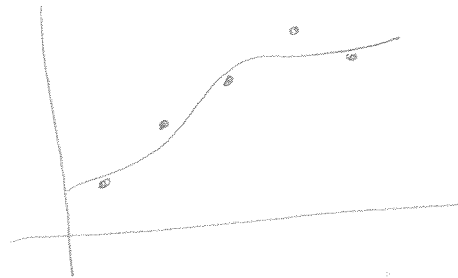
GROUP NAME: <u>CCS</u>	Student Names (First and Last)
Logo:	Speaker/Presenter: <u>Corneal Douglas</u>
Date: <u>08/28/13</u>	Writer/Prep: <u>Courtney Grubb</u>
Topics:	QC/Leader: <u>Stephen Smith</u>

Instructions: average rate of change  
Apple stock prices over the last 5 yrs.

Data:

YRS.	Stock Price
'09	\$211.98
'10	\$336.12
'11	\$405.00
'12	\$532.17
'13	\$501.02

graph



$$\frac{\Delta Y}{\Delta X} = \frac{433.3 - 492.69}{-1} = 59.4$$

59.4 stock price/yr

in 2010 the instantaneous rate of change was 117 \$ per year

GROUP NAME: <u>The factors</u>	Student Names (First and Last)
Logo:	Speaker/Presenter: <u>Ryan Bigley</u>
Date: <u>4/29</u>	Writer/Prep: <u>Kevin Chavez</u>
Topics:	QC/Leader: _____

Instructions: Average rate of change  
instantaneous rate of change

Y	X
<del>1950</del>	3.5
1960	2.2
1970	1.85
1980	2.5
1990	6.7
2000	9.2

$$m = \frac{\Delta Y}{\Delta X} = \frac{2000 - 1950}{9.2 - 3.5} = \frac{50 \text{ years}}{5.7 \text{ billions}}$$

1,140,000,000  
 Dollars per year

in 1965 the instantaneous rate of change was 1.2 million dollars

in 1980 the instantaneous rate of change was 2.1 million dollars

GROUP NAME: <u>Time is Money</u>	Student Names (First and Last)
Logo:	Speaker/Presenter: <u>Angelika Morvek</u>
Date: <u>08/28/13</u>	Writer/Prep: <u>Shivam Singh</u>
Topics:	QC/Leader: <u>Eugenie Pelaez</u>

Instructions:

Price of iPhone 4S changes.

L1	L2
0	700
1	600
2	350
3	200
4	650
5	250

**STAT** 1: EDIT

**STAT**  $\rightarrow$  CALC 4: LIN Reg.

$\downarrow$   
**Y=** **VARS** 5:  $\rightarrow$   $\rightarrow$  <enter>

**STAT**  $\rightarrow$  CALC 5: QUAD Reg.  
 <enter>

**Y2=** **VARS** 5:  $\rightarrow$   $\rightarrow$  <enter>

SPLIT

Y = BOTTOM of Linear / ( $x \leq 3.5$ )

Y2 = BOTTOM of QUAD / ( $x \geq 3.5$ )

Average Change:

$$0 \quad 619.05$$

$$5 \quad 375$$

$$\frac{\Delta Y}{\Delta x} = \frac{375 - 619.05}{5 - 0}$$

$$= -48.81$$

\$ per year

Instantaneous Rate of Change

CALC = **2nd** **TRACE** 6: dy/dx

$$x = 2.5$$

$$dy/dx = (-64.28571)$$

In 2.5 years price decreases

by \$ -64.28571

per year

GROUP NAME: <u>The Scientists</u>	Student Names (First and Last)
Logo:	Speaker/Presenter: <u>Darin Cioatisan</u>
Date: <u>8/28/13</u>	Writer/Prep: <u>Nicole Rowall</u>
Topics: <u>The number of extinct species</u>	QC/Leader: <u>Kiersten Hendrickson</u>

Instructions:

$L_1$	$L_2$
2008	750
2009	836
2010	844
2011	870
2012	876
2013	908

$$\frac{\Delta y}{\Delta x} = \frac{2013 - 2008}{908 - 750} = \frac{158}{5} = 31.6 \text{ species/yr}$$

AT year 2010 the instantaneous Rate is  $dy/dx = 14.4 \text{ species/yr}$