

Course Number Course Title Credits
MAT 208 Linear Algebra 4

Hours: Co- or Pre-requisite Implementation
Lecture/Lab/Other Semester & Year

4/0/0 Pre-reg: MAT 151 Spring 2022

Catalog description:

An introduction to linear algebra topics including linear equations and matrices, determinants, independence and basis, vector spaces and subspaces, the four fundamental subspaces, orthogonality, linear transformations and eigenvalues and eigenvectors. Applications of linear algebra are included.

General Education Category: Course coordinator:

Goal 2: Mathematics Betty Peterson, petersob@mccc.edu, 609-570-3421

Required texts & Other materials:

Elementary Linear Algebra, Ron Larson, 8th Edition ISBN #: 978-1-305-65800-4 OR

The instructor may select a comparable book.

Graphing Calculator or computer utility may be required

Course Student Learning Outcomes (SLO):

Upon successful completion of this course, the student will be able to:

- 1. Solve systems of linear equations and interpret their results. (ILGs 2, 11; PLOs 1 4)
- 2. Describe properties of linear systems using vectors. (ILGs 2, 11; PLOs 1 4)
- 3. Demonstrate an understanding of linear transformations. (ILGs 2, 11; PLOs 1 4)
- 4. Perform and interpret matrix operations. (ILGs 2, 11; PLOs 1 4)
- 5. Compute and interpret determinants of matrices. (ILGs 2, 11; PLOs 1 4)
- 6. Demonstrate an understanding of vector spaces and subspaces. (ILGs 2, 11; PLOs 1 4)
- 7. Demonstrate an understanding of eigenvalues and eigenvectors. (ILGs 2, 11; PLOs 1 4)

Course-specific Institutional Learning Goals (ILG):

Institutional Learning Goal 2. Mathematics. Students will use appropriate mathematical and statistical concepts and operations to interpret data and to solve problems.

Institutional Learning Goal 11. Critical Thinking: Students will use critical thinking skills understand, analyze, or apply information or solve problems.

Program Learning Outcomes for Mathematics AS (PLO)

- 1. Apply a range of mathematical skills spanning fundamental concepts to more advanced mathematical concepts.
- 2. Apply quantitative knowledge, including the required technological skills and theoretical knowledge.
- 3. Demonstrate critical thinking skills to solve real world problems using mathematical modeling.
- 4. Communicate methods of solutions and results to problems using mathematical language and notation.

<u>Units of study in detail – Unit Student Learning Outcomes:</u>

<u>Unit I</u> [Linear Equations and Matrices] [Supports Course SLO #1, 4]

Learning Objectives

The student will be able to:

- Determine whether a system of linear equations is consistent or inconsistent.
- Use matrices and Gaussian elimination with back-substitution to solve a system of linear equations.
- Use matrices and Gauss-Jordan elimination to solve a system of linear equations.
- Perform operations on matrices: add, subtract, multiply and multiply by a scalar
- Use the properties of matrices: addition, scalar multiplication, and matrix multiplication. As well as zero matrices, inverse matrices, and the identity matrix.
- Find the transpose and the inverse (if it exists) of a matrix.
- Factor a matrix into a product of elementary matrices.
- Find and use an *LU*-factorization of a matrix to solve a system of linear equations.
- Find the least squares regression line for a set of data.

<u>Unit II</u> [Determinants] [Supports Course SLO #5]

Learning Objectives

The student will be able to:

- Find the determinant of a 2 x 2 matrix and the minors, and cofactors of any matrix.
- Use expansion by cofactors to find the determinant of a matrix.
- Find the determinant of a triangular matrix.
- Use elementary row and column operations to evaluate a determinant.
- Find the determinant of a matrix product and scalar multiple of a matrix.
- Find the determinant of an inverse matrix and recognize equivalent conditions for a nonsingular matrix.
- Find the determinant of a transpose matrix.
- Use Cramer's Rule to solve a system of n linear equations in n variables.
- Apply determinants to find the area, volume, and equations of lines and planes.

<u>Unit III</u> [Vector Spaces] [Supports Course SLOs #2, 6]

Learning Objectives

The student will be able to:

- Perform basic vector operations in Rⁿ.
- Write a vector as a linear combination of other vectors.
- Understand the definition of a vector space and recognize why a given set would not be a vector space.
- Determine whether a subset *W* of a vector space *V* is a subspace of *V*.
- Determine subspaces of R^n .
- Write a linear combination of a set of vectors in a vector space *V*.
- Determine whether a set S of vectors in a vector space V is a spanning set of V.

- Determine whether a set of vectors in a vector space V is linearly independent.
- Recognize bases in the vector spaces R^n , P_n , $M_{m,n}$.
- Find the dimension of a vector space.
- Find a basis for the row space, a basis for the column space, and the rank of a matrix.
- Find the nullspace of a matrix.
- Find the solution of a consistent system Ax = b in the form of $x = x_p + x_h$.
- Find a coordinate matrix relative to a basis in Rⁿ.

<u>Unit IV</u> [Inner Product Spaces] [Supports Course SLO #6]

Learning Objectives

The student will be able to:

- Find the length of a vector and find a unit vector.
- Find a dot product and the angle between two vectors, determine orthogonality, and verify the Cauchy-Schwarz Inequality.
- Find the inner product of two vectors in \mathbb{R}^n , P_n , $M_{m,n}$, and $\mathbb{C}[a, b]$.
- Find an orthogonal projection of a vector onto another vector in an inner product space.
- Show a set of vectors is orthogonal and forms an orthonormal basis, and represent a vector relative to an orthonormal basis.
- Apply the Gram-Schmidt orthonormalization process.
- Find the orthogonal complement of a subspace and the projection of a vector onto a subspace.
- Find the four fundamental subspaces of a matrix.
- Find the cross product of two vectors in R³.

<u>Unit V</u> [Eigenvalues and Eigenvectors] [Supports Course SLO #7]

Learning Objectives

The student will be able to:

- Verify eigenvalues and corresponding eigenvectors.
- Find eigenvalues and corresponding eigenspaces.
- Use the characteristic equation to find eigenvalues and eigenvectors.
- Find the eigenvalues and eigenvectors of a linear transformation.
- Find the eigenvalues of similar matrices, determine whether a matrix *A* is diagonalizable, and find a matrix *P* such that *P*¹*AP* is diagonal.
- Recognize, and apply properties of, symmetric matrices.
- Recognize, and apply the properties of, orthogonal matrices.

<u>Unit VI</u> [Linear Transformations] [Supports Course SLO #3]

Learning Objectives

The student will be able to:

- Demonstrate a function is a linear transformation and find a linear transformation.
- Find the kernel of a linear transformation.
- Find a basis for the range, the rank, and the nullity of a linear transformation.
- Determine whether a linear transformation is one-to-one or onto.
- Find the standard matrix for a linear transformation.
- Find the standard matrix for the composition of linear transformations and find the inverse of an invertible linear transformation.

Evaluation of student learning:

Students will be assessed informally by completing assigned homework and formally by quizzes and exams. Selected homework problems ensure students are meeting the learning goals described above in detail. All selected quiz and test questions assess each of the course goals.

Suggested grade distribution:

Midterm Exam(s) 20% Final Exam 30% Graded Assignments or quizzes 50%

Refer to the section syllabus for the exact distribution and explanation.