Solving Modular Problems

Chapter 10 – Section 3

Simple Modular Equations

- The solution to a modular equation is a set of all numbers in one or more equivalence classes.
- It is possible that the solution is the empty set.
- A simple modular equation is of the nature $x \equiv a \pmod{m}$

No in-class assignment problem

Solving a Simple Equation

- $x \equiv 5 \pmod{6}$
- 5 is the name of an equivalence class in mod 6.
- The solution is the equivalence class 5.
- $x = \{5, 11, 17, 23, \ldots\}$
- Remember each to find the next number in an equivalenc class add the mod.

- $x \equiv 25 \pmod{4}$
- 25 is not less than 4.
- Change 25 to the name of the equivalence class 25 is in. $25 \equiv 1 \pmod{4}$
- $x = \{1, 5, 9, 13, \ldots\}$

In-class Assignment 25 - 1

Solving More Complicated Modular Equations

- $5x \equiv 17 \pmod{4}$
- Change 17 to the name of the class it belongs to. $17 \equiv 1 \pmod{4}$
- Write 4 statements because the mod is 4 replacing x with the name of an equivalence class.
  - $5 \cdot 0 \equiv 1 \pmod{4}$ False
  - $5 \cdot 1 \equiv 1 \pmod{4}$ True
  - $5 \cdot 2 \equiv 1 \pmod{4}$ False
  - $5 \cdot 3 \equiv 1 \pmod{4}$ False
  - The solution is equivalence class 1.
  - $x = \{1, 5, 9, 13, \ldots\}$

In-class Assignment 25 - 2

Another Equation to Solve

- $2x \equiv 4 \pmod{8}$
- Equivalence classes 2 and 6 are solutions.
- $x = \{2,10,18, \ldots\} \cup \{6, 14, 22, \ldots\}$
- Notice the sets can be joined as one set.
- $x = \{2, 6, 10, 14, \ldots\}$

$2 \cdot 0 \equiv 4 \pmod{8}$ False
$2 \cdot 1 \equiv 4 \pmod{8}$ False
$2 \cdot 2 \equiv 4 \pmod{8}$ True
$2 \cdot 3 \equiv 4 \pmod{8}$ False
$2 \cdot 4 \equiv 4 \pmod{8}$ False
$2 \cdot 5 \equiv 4 \pmod{8}$ False
$2 \cdot 6 \equiv 4 \pmod{8}$ True
$2 \cdot 7 \equiv 4 \pmod{8}$ False

In-class Assignment 25 - 2

A Word Problem

- Ann wants to see a play. The date of the play is 65 days from today. Ann is a nurse and her schedule is 5 – 10 hour days on and 3 days off. Will she be off on the day of the play if she is in the second day of her 5 days on?

- On = 1 and Off = 2
- 8 day schedule
- 1 1 1 1 1 2 2 2
- Remainder =1
- 1 day past 2nd day of schedule.

$$8\overline{)65}\quad 8 \text{ r } 1$$

Ann will be on.

In-class Assignment 25 - 3

Symbols of Inequalities

- < means "less than."
- ≤ means is "less than or equal to" or "no more than."
- > means "greater than."
- ≥ means "greater than or equal to" or "no less than."

No in-class assignment problem

Solving Restricted Modular Equations

- Find the set of whole number solutions given
- that
  $x < 40$
  $x \equiv 1 \pmod 7$
  $x \equiv 4 \pmod 5$
  $x \equiv 5 \pmod 8$
- Find all the solution sets for the 3 equations.
  - x = {1, 8, 15, 22, 29, 36}
  - x = {4, 9, 13, 17, 21, 25, 29, 33, 37}
  - x = {5, 13, 21, 29, 37}
- Note – only numbers < 40.
- Find the intersection of the solution sets.
- x = 29

In-class Assignment 25 - 4

Solving a Verbal Problem in Mod Systems

- The football team has 50 members. When the coach put them in drill teams of 7 there were 4 players left. When the coach had them in drill teams of 3 all players were used. When the coach put them in teams of 6 there were 3 players left. How many were at practice on this day?
- Mods are the groupings
- Numbers are whats left.

$x \le 50$
$x \equiv 4 \pmod 7$
$x \equiv 0 \pmod 3$
$x \equiv 3 \pmod 6$

- Solve as before.
- 39 players were at practice on this day.

In-class Assignment 25 – 5, 6

A Short Cut

- $x < 40$
  $x \equiv 1 \pmod 7$
  $x \equiv 4 \pmod 5$
  $x \equiv 5 \pmod 8$
- Find the solution set for the largest mod.
  x = {5, 13, 21, 29, 37}
- Cross out any number that **does not** leave a remainder of 1 when divided by 7 (5, 13, 21, 37 crossed out) or a remainder of 4 when divided by 5 (the same numbers crossed out).
- x = {29}

In-class Assignment 25 – 4 (short cut)