COURSE OUTLINE

Course Number       Course Title       Credits
MAT208             Linear Algebra       4

Hours:            Co- or Pre-requisite
lecture/Lab/Other  Calculus I
4 lecture hours

Implementation
sem/year
Fall 2012

Catalog description (2012-2014 Catalog): Designed as a sophomore level course for majors in Computer Science, Engineering, Physics, Biology, Chemistry or Mathematics. Topics include systems of linear equations from a computational as well as a theoretical point of view, the geometry of \( \mathbb{R}^n \), linear equations and matrices, determinants, independence and basis, vector spaces and subspaces, the four fundamental subspaces, orthogonality, linear transformations, and eigenvalues and eigenvectors. Applications to engineering, statistics, economics, science and other areas will be included and \( MATLAB \) will be used to gain additional insights into the concepts of linear algebra.

Is course New, Revised, or Modified? Revised 2014

Required texts/other materials:
Elementary Linear Algebra with Applications, Kolman and Hill, 9th Edition
Publisher: Pearson Prentice-Hall
\( MATLAB \) Software or Graphing Calculator

Revision date: Course coordinator: Richard Porter
Fall 2014 Phone: 609.570.3826
Email: porterr@mccc.edu

Information resources: The Mercer County Community College Library has reference books that students may use. Students are also encouraged to utilize the Learning Centers for additional resources and/or tutoring.

Other learning resources: A list of freeware and other supplemental course materials that students may find helpful is available at www.mccc.edu/~porterr on the MAT208 homepage.
Course-specific General Education Knowledge Goals and Core Skills:

General Education Knowledge Goals

Goal 1. Communication. Students will communicate effectively in both speech and writing.
Goal 2. Mathematics. Students will use appropriate mathematical and statistical concepts and operations to interpret data and to solve problems.
Goal 4. Technology. Students will use computer systems or other appropriate forms of technology to achieve educational and personal goals.

MCCC Core Skills

Goal A. Written and Oral Communication in English. Students will communicate effectively in speech and writing, and demonstrate proficiency in reading.
Goal B. Critical Thinking and Problem-solving. Students will use critical thinking and problem solving skills in analyzing information.
Goal D. Information Literacy. Students will recognize when information is needed and have the knowledge and skills to locate, evaluate, and effectively use information for college level work.
Goal E. Computer Literacy. Students will use computers to access, analyze or present information, solve problems, and communicate with others.

In the Course Competencies/Goals list, General Education Knowledge Goals will be denoted GE and MCCC Core Skills will be denoted CS.

Course Competencies/Goals:

Students will demonstrate the ability to:

1. generalize the properties of vectors to n-space. (GE 2, CS B, E)
2. solve \(Ax=b\) for matrices by the method of elimination involving row-reduction, pivots, back-substitution, the invertibility of a square matrix \(A\) and matrix factorization. (GE 1, 2, 4, CS A, B, E)
3. define a vector space and subspace. (GE 1, 2, CS A, B)
4. use the properties of determinants in applications involving the inverse of a matrix and volume problems. (GE 1, 2, 4, CS A, B, E)
5. define and apply the concepts of basis, dimension and linear independence and determine bases for the four fundamental subspaces. (GE 1, 2, CS A, B)
6. apply projections for least-square solutions. (GE 1, 2, 4, CS A, B, E)
7. orthogonalize a matrix by Gram-Schmidt factorization. (GE 1, 2, 4, CS A, B, E)
8. apply the algebra of linear transformations. (GE 1, 2, 4, CS A, B, E)
9. construct the Singular Value Decomposition to diagonalize square and rectangular matrices. (GE 1, 2, 4, CS A, B, E)
10. calculate and use eigenvalues and eigenvectors in diagonalization of a matrix and in computing powers of a matrix \(A\). (GE 1, 2, 4, CS A, B, D, E)
11. calculate the pseudoinverse of a non-square matrix. (GE 1, 2, 4, CS A, B, D, E)
12. complete projects involving practical applications of linear algebra. (GE 1, 2, 4, CS A, B, D, E)

The main learning goals of this Linear Algebra course may be summarized by saying that this course should give students a sound foundation in Linear Algebra that will serve them well in future courses, continue the development of the mathematical maturity of the students, and introduce them to the use of technology tools to tackle more difficult but applications-oriented problems.
In the following Units of Study in Detail, Course Competencies/Goals will be denoted Course Goals.

**Units of study in detail.**

1. **LINEAR EQUATIONS AND MATRICES**
   
   **(2 weeks)**

   At the end of Unit 1, the student should be able to:
   
   - calculate the dot product between vectors. (Course Goal 1)
   - calculate the length of a vector. (Course Goal 1, Gen Ed Goal 2)
   - explain the concept of orthogonality of vectors. (Course Goal 1)
   - create a unit vector from a given vector. (Course Goal 1)
   - demonstrate that one vector is a linear combination of given vectors. (Course Goal 1)
   - apply matrix operations. (Course Goal 1)
   - set up matrix operations using proper technology. (Course Goal 1, 12)

2. **DETERMINANTS AND INVERSES**
   
   **(2 weeks)**

   At the end of Unit 2, the student should be able to:
   
   - calculate the inverse of a matrix. (Course Goal 2, 12)
   - solve systems by using row reduction and LU factorization. (Course Goal 2, 12)
   - recognize the connection between the elimination process and factoring a matrix. (Course Goal 2)
   - define determinant. (Course Goal 10)
   - calculate the determinant of a square matrix and interpret it in terms of invertibility of a matrix. (Course Goal 4)
   - apply the basic properties of determinants. (Course Goal 4, 12)
   - apply Cramer’s Rule to solve systems of equations and volume problems. (Course Goal 4, 12)

3. **VECTOR SPACES AND SUBSPACES**
   
   **(3.5 weeks)**

   At the end of Unit 3, the student should be able to:
   
   - explain the defining properties of a vector space. (Course Goal 3)
   - construct examples of vector spaces. (Course Goal 3)
   - explain why a set defined with the necessary operations is or is not a vector space. (Course Goal 3)
   - explain why a subset of a given vector space is or is not a subspace. (Course Goal 3)
   - define the span of a set of vectors. (Course Goal 3)
   - determine if a collection of vectors from a given vector space is a spanning set for the vector space. (Course Goal 3, 12)
   - define linear independence. (Course Goal 3 and 5)
   - calculate whether or not a given set of vectors is linearly independent. (Course Goal 5, 12)
   - calculate the rank of a given matrix. (Course Goal 5, 12)
   - explain the defining properties of a basis for a vector space. (Course Goal 5)
   - determine if a given set of vectors from a vector space is or is not a basis for the space. (Course Goals 5, 12)
   - explain what is meant by the dimension of a vector space. (Course Goal 5)
• state the properties of subspaces and the relationships among the four fundamental subspaces of a matrix. (Course Goals 3 and 5, Gen Ed Goals 1 and 2)
• explain why the equation $Ax=b$ is consistent if and only if $b$ is in the column space of $A$. (Course Goals 3 and 5, Gen Ed Goals 1 and 2)
• discuss how linear independence, spanning sets, basis and dimension are related. (Course Goals 5 and 12)

4. ORTHOGONALITY AND LEAST SQUARES  
(2 weeks)

At the end of Unit 4, the student should be able to:
• state the properties of orthogonal matrices. (Course Goal 7)
• derive the normal equations for a least squares problem and solve it. (Course Goal 7)
• explain what condition must be satisfied for the normal equations to have a unique solution. (Course Goal 6)
• apply least squares approximations to problems to minimize errors. (Course Goals 6 and 12)
• calculate the error in a least squares problem. (Course Goals 6 and 12)
• apply the Gram-Schmidt process to construct an orthonormal set of vectors. (Course Goal 7, 12)
• construct the QR factorization of a matrix. (Course Goal 7 and 12)
• apply the QR factorization to least-squares problems. (Course Goals 6, 7, and 12)
• compare the methods of matrix factorization studied so far. (Course Goals 2 and 7)

5. EIGENVALUES AND EIGENVECTORS  
(3 weeks)

At the end of Unit 5, the student should be able to:
• define eigenvalue and eigenvector. (Course Goal 10)
• calculate the characteristic equation of a square matrix and solve to find eigenvalues. (Course Goal 10, 12)
• calculate the associated eigenvectors of a square matrix. (Course Goal 10, 12)
• explain how the eigenvalues of similar matrices are related. (Course Goals 4 and 10)
• analyze the relationship among determinants and number of eigenvalues, determinants and the product of eigenvalues, and the trace and the sum of eigenvalues. (Course Goals 4, 10, 12)
• explain the application of diagonalizing a matrix to the matrix exponential. (Course Goal 9)
• define pseudoinverse for a non-square matrix. (Course Goal 11)
• construct the singular value decomposition of an $m \times n$ matrix. (Course Goal 9, 12)
• explain the relationships among the columns of $U$ and $V$ of $A = U \Sigma V^T$, the singular value decomposition of an $m \times n$ matrix $A$, and the four fundamental subspaces associated with $A$. (Course Goals 5, 9, 10, 12)
• summarize the use of the singular value decomposition in applications such as Web search engines and image processing. (Course Goal 9, 12)
• apply the singular value decomposition and pseudoinverse to solve least squares problems. (Course Goals 9, 11, and 12)
• conclude, through the use of technology, that the singular value decomposition is necessary for factoring non-square matrices. (Course Goals 9 and 12)
6. GENERALIZED VECTOR SPACES AND LINEAR TRANSFORMATIONS  (2.5 weeks)

At the end of Unit 6, the student should be able to:

- state the definition of a linear transformation from a vector space V to another vector space W. (Course Goal 8)
- give examples of linear transformations. (Course Goal 8)
- calculate the kernel and range of a linear transformation (Course Goal 8)
- identify a linear transformation and find and use its matrix representation. (Course Goal 8,12)
- illustrate the process of change-of-basis by building on previous work and definitions. (Course Goal 8)
- calculate the matrix representation of a linear transformation from a vector space V to a vector space W with respect to two given bases. (Course Goal 8)
- examine the geometry of linear transformations. (Course Goal 8)

Evaluation of Student Learning

Students will receive regular feedback on their work through assignments, examinations, lab work, and projects. The syllabus for this course should describe the schedule for these assessment tools and how they will be used to calculate grades. Learning activities will consist of a combination of lectures, lab work and computer assignments. The specific choices for assessment will rest with the instructor. Outside of class, students are expected to do a significant amount of work to achieve learning goals for this course. A typical grading scheme for this course follows:

<table>
<thead>
<tr>
<th>Component</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>Exams</td>
<td>30%</td>
</tr>
<tr>
<td>Project</td>
<td>15%</td>
</tr>
<tr>
<td>Graded Assignments</td>
<td>15%</td>
</tr>
<tr>
<td>Computer Labs</td>
<td>10%</td>
</tr>
<tr>
<td>Final Exam</td>
<td>30%</td>
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</tbody>
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Academic Integrity Statement:

Under no circumstance should students knowingly represent the work of another as one’s own. Students may not use any unauthorized assistance to complete assignments or exams, including but not limited to cheat-sheets, cell phones, text messaging and copying from another student. Violations should be reported to the Academic Integrity Committee and will be penalized. Please refer to the Student Handbook for more details.